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A LIST OF "GIMMICKS" FOR USE WITH JUNIOR HIGH SCHOOL STUDENTS.

CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT

PUB DATE

67

EDRS PRICE MF-\$0.50 HC-\$3.96 97P.

DESCRIPTORS- *INSTRUCTION, *LOW ACHIEVERS, *RESOURCE MATERIALS, *SECONDARY SCHOOL MATHEMATICS, ARITHMETIC, GRADE 7, GRADE 8, GRADE 9, LEARNING, MATHEMATICS, SLOW LEARNERS, CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT,

THIS PAPER, CONSISTING OF A COLLECTION OF MATHEMATICS TEACHING IDEAS AND STUDENT ACTIVITIES, IS A COMPILATION OF THOSE FOUND TO BE MOST EFFECTIVE BY THE CONTRIBUTING TEACHERS IN THE 1967-68 CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT. THE ACTIVITIES DESCRIBED IN THIS PAPER ARE INTENDED FOR JUNIOR HIGH SCHOOL STUDENTS WITH VARYING DEGREES OF COMPETENCE IN MATHEMATICS, INCLUDING THOSE STUDENTS CLASSIFIED AS LOW ACHIEVERS. (RP)

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A LIST OF "GIMMICKS"
FOR USE WITH JUNIOR HIGH SCHOOL STUDENTS

COMPILED BY MEMBERS OF THE 1967-68
CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT

1164 26th Street
Des Moines, Iowa 50311

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Projects

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F. Johnson

Days in a Month

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C. Lippert

Numerical Tic-Tac-Toe

D. O'Neil

Card Sorting via Base Two

D. O'Neil

The Miser

D. O'Neil

Stepping through Paper

D. O'Neil

The Only Way to Rent

D. O'Neil

GAMES FOR MATH STUDENTS

	<u>Cost</u>
1. Euclid, 92 Geometric Puzzle Games -- #113	\$1.00
2. Hi-Q -- #120	\$1.00
3. It's Hexed -- #115	\$1.00
4. Pythagorus, 179 Puzzle Games -- #125	\$1.00
5. Tormentor -- #116	\$1.00
6. Voodoo, 140 Puzzle Games -- #112	\$1.00

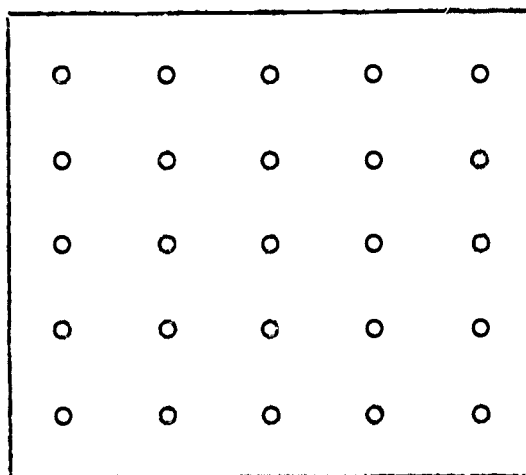
All of the above games can be procured from either --

L. W. Holley & Sons Co. Tel. No. 243-6247
100 East Grand Avenue
Des Moines, Iowa

or

Kohner Bros., Inc.
Tryne Game Division
New York, New York 10012

7. Reckoner Puzzle



Consists of a square block of wood approximately 5 x 5 inches with holes as indicated to the left. There are 25 pegs, five numbered 1, five numbered 2, five numbered 3, etc. The object is to arrange the pegs so that the sum of each row and column as well as the main diagonals is 15.

THE DAY OF THE WEEK

PURPOSE

To introduce an attention getting algorithm, in use many years ago, which was used to calculate the day of the week any date for three thousand years from the year one falls on.

METHOD

To the last two digits of the given year add $\frac{1}{4}$ of those digits. Disregard any fractional parts. To this sum add the day of the month and the index of the month and century. The latter two numbers can be found in the charts below. Finally, divide the above result by 7. The remainder is the day of the week, counting Sunday the 1st, Monday the 2nd, etc.

INDEX OF MONTHS		CENTENNIAL INDEX						Index
June	0	200	900	1800	2200	2600	3000	0
Sept. & Dec.	1	300	1000					6
April & July	2	400	1100	1900	2300	2700		5
Jan. & Oct.	3	500	1200	1600	2000	2400	2800	4
May	4	600	1300					3
Aug.	5	000	700	1400	1700	2100	2500	2
March, Feb. & Nov. ...	6	100	800	1500				1

Note: In leap years the index of Jan. is 2 and the index of Feb. is 5. There are no other changes.

Examples

I. Independence was declared July 4, 1776; on what day of the week did this occur?

Solution -- 76 (last 2 digits of given year)
 19 ($\frac{1}{4}$ of last 2 digits)
 4 (day of month)
 2 (index for July)
 2 (index for century)
103 Sum

$103 \div 7$ gives remainder of 5

Hence, July 4, 1776, was a Thursday.

II. President Lincoln was assassinated on April 14, 1865. What day of the week was he killed?

Solution -- 65 (last 2 digits of given year)
16 (1/4 of last 2 digits) (Note the remainder is dropped)
14 (day of month)
2 (index of April)
0 (index of century)
97 Sum .

$97 \div 7$ gives a remainder of 6

Hence, Lincoln was killed on a Friday.

One final comment; if the remainder upon division by 7 is 0, the day of the week is Saturday.

REFERENCE

Woodward, R. J. PRACTICAL ARITHMETIC. Bechtold Printing & Book Manufacturing Co., St. Louis, Missouri, 1894

The Pixilated Deliveryman Stories

- What is it - A suggested procedure to help introduce negative integers and the operations of addition and subtraction with positive and negative integers.
- Why use it - To create interest by using a psuedo-practical situtation.
- When to use it - Any time, but once the negative integers are introduced the concept should not be allowed to decay through lack of use.
- How to use it - Students are to think of themselves as operators of a business, having checks and bills delivered to them by a very inefficient deliveryman. The operator of the business realizes the vagaries of the deliveryman so he does not cash checks nor pay bills but keeps them in his safe. He has developed a method of record keeping and this is what we will study.
- (One could prepare a set of "bills" of different denominations. The checks could be on green or white paper and the bills on red paper. Two students could "act out" the process of delivering and picking up checks and bills.)
- The following wording is suggested for use with the students.
- If you are presented a check for \$10, does this make you richer or poorer? By how much? We will record this as +10.
- (Note the raised symbols for the "Quality" of the number, as opposed to the use of + and - to indicate operations.)
- If you are presented a bill for \$7, does this make you richer or poorer? By how much? How can we record this? (Answer: -7)
- If a check of \$8 that you had previously received was in error and must be returned, does this make you richer or poorer? By how much? We will record this as -+8.

The Pixilated Deliveryman Stories (continued)

If a bill of \$6 that you had previously received was in error and must be returned, does this make you richer or poorer? By how much? (Answer: \$6 richer.) How can we record this? (Answer: -6 .) Is this equivalent to receiving a check of \$6? (Answer: Yes.) If so, then $-6 = +6$.

(This "double negative" in mathematics could be related to the double negative in English. For example, "I haven't none" means I have some.)

Suppose you are given a check for \$4 and a bill for \$2. Are you richer or poorer? By how much? How do you record this transaction?

$$+4 + -2 = +2$$

Suppose you are given a check for \$3 and a check for \$11. Are you richer or poorer? By how much? How do you record this transaction?

$$+3 + 11 = +14$$

Suppose you are given a check for \$8 and a bill for \$2 is retrieved. Are you richer or poorer? By how much? How do you record this transaction?

$$+8 - 2 = +10, \text{ or } +8 + +2.$$

(It is possible to "retrieve" first and "give" last, but slightly awkward. See the next example.)

Suppose that two bills are retrieved, one for \$6 and one for \$7. Are you richer or poorer? By how much? How do you record this transaction?

$$-6 - 7 = +13, \text{ or } +6 + +7.$$

We leave it to the teacher's ingenuity to devise problems for this unit, however, a few "skill" problems are suggested on the next page.

ANSWERS TO "DELIVERYMAN" STORIES

1) $+8$	6) $+5$	1) $+18$	6) $+10$	1) $+4$	6) $+17$
2) $+3$	7) $+7$	2) $+3$	7) $+18$	2) $+19$	7) $+11$
3) $+9$	8) $+2$	3) $+8$	8) $+5$	3) $+34$	8) $+35$
4) $+4$	9) $+14$	4) $+20$	9) $+28$	4) $+39$	9) $+22$
5) $+6$	10) $+19$	5) $+9$	10) $+24$	5) $+21$	10) $+45$

1) $+13$	6) $+40$	1) $+30$	6) -29	1) $+49$	6) $+70$
2) $+15$	7) $+40$	2) $+6$	7) $+34$	2) $+15$	7) -23
3) $+20$	8) $+27$	3) $+58$	8) $+74$	3) $+26$	8) -31
4) $+25$	9) -3	4) -6	9) -26	4) $+60$	9) $+15$
5) $+8$	10) $+8$	5) $+3$	10) $+75$	5) $+50$	10) $+70$

1) $+98$	6) $+49$	1) $+89$	6) $+65$	1) -2	6) -58
2) $+19$	7) $+63$	2) $+13$	7) $+100$	2) -24	7) $+8$
3) $+34$	8) $+47$	3) -10	8) $+96$	3) -7	8) $+25$
4) $+46$	9) $+80$	4) -76	9) -20	4) -2	9) $+125$
5) $+29$	10) $+100$	5) -25	10) -33	5) $+78$	10) $+86$

1) -16	6) $+115$
2) $+32$	7) $+42$
3) $+90$	8) $+76$
4) $+69$	9) -80
5) $+84$	10) -100

SIGNED ALGEBRAS

No. 3		No. 4	
1)	$+6 + -2 =$	1)	$+11 + +2 =$
2)	$+16 - -3 =$	2)	$+12 - -3 =$
3)	$+18 + +16 =$	3)	$+16 + +4 =$
4)	$+22 + +17 =$	4)	$+22 - -3 =$
5)	$+25 - +4 =$	5)	$+14 + -6 =$
6)	$+14 - -3 =$	6)	$+27 + +13 =$
7)	$+15 + -4 =$	7)	$+14 + +26 =$
8)	$+25 + +10 =$	8)	$+12 - -15 =$
9)	$+26 - +4 =$	9)	$+12 + -15 =$
10)	$+42 - -3 =$	10)	$+15 + -7 =$

No. 5			
1)	$+15 + +15 =$	5)	$+35 + +14 =$
2)	$+22 + -16 =$	2)	$+27 + -12 =$
3)	$+41 + +17 =$	3)	$+42 + -16 =$
4)	$+16 + -22 =$	4)	$+45 - -15 =$
5)	$-72 + +75 =$	5)	$+62 + -12 =$
6)	$-45 + +16 =$	6)	$+46 + +24 =$
7)	$+47 + -13 =$	7)	$+12 + -35 =$
8)	$+50 + +24 =$	8)	$-15 + -16 =$
9)	$-50 + +24 =$	9)	$-25 + +40 =$
10)	$+50 - -25 =$	10)	$-15 + +85 =$

SIGNED NUMBERS

No. 7

- 1) $+52 + +46 =$
- 2) $+31 + -12 =$
- 3) $+46 - +12 =$
- 4) $+43 - -3 =$
- 5) $+45 + -16 =$
- 6) $+55 - +6 =$
- 7) $+48 - -15 =$
- 8) $+49 - +2 =$
- 9) $+63 + +17 =$
- 10) $+81 - -19 =$

No. 8

- 1) $+75 + -10 =$
- 2) $+26 + -3 =$
- 3) $+42 + -52 =$
- 4) $-63 + -13 =$
- 5) $-50 - -25 =$
- 6) $+75 + +14 =$
- 7) $+86 + +14 =$
- 8) $+12 + +84 =$
- 9) $+14 + -34 =$
- 10) $+62 + -95 =$

No. 9

- 1) $+14 - +16 =$
- 2) $+12 + -36 =$
- 3) $+55 - +62 =$
- 4) $+43 - +45 =$
- 5) $+66 - -12 =$
- 6) $-42 + -16 =$
- 7) $-67 + +75 =$
- 8) $+75 + -50 =$
- 9) $+75 - -50 =$
- 10) $+62 - -24 =$

No. 10

- 1) $+16 + -32 =$
- 2) $+18 + +14 =$
- 3) $+77 - -13 =$
- 4) $+85 - +16 =$
- 5) $+72 - -12 =$
- 6) $+100 + +15 =$
- 7) $+76 + -34 =$
- 8) $+14 - -62 =$
- 9) $-15 + -65 =$
- 10) $-12 + -88 =$

SIGNED NUMBERS

No. 11	No. 12
1) $+55 + -15 =$	1) $+126 + +82 =$
2) $+65 + +45 =$	2) $+79 + -46 =$
3) $+75 - +35 =$	3) $+124 + -29 =$
4) $+85 - +45 =$	4) $+149 - +42 =$
5) $+95 - -25 =$	5) $+167 + +43 =$
6) $+105 + -25 =$	6) $+147 - -23 =$
7) $+115 - -35 =$	7) $+256 + -67 =$
8) $+125 - +65 =$	8) $+175 + -25 =$
9) $+145 - -25 =$	9) $+175 - -50 =$
10) $-135 - -65 =$	10) $+223 - +46 =$

No. 13	No. 14
1) $+351 + -26 =$	1) $+a + +a =$
2) $+248 - -42 =$	2) $+a + +3a =$
3) $+375 + +125 =$	3) $+a + -2a =$
4) $+369 + -42 =$	4) $+a - +3a =$
5) $+496 + +24 =$	5) $+2a + +4a =$
6) $+672 + -145 =$	6) $+a - -5a =$
7) $+355 + +75 =$	7) $+3a + -2a =$
8) $+422 - -122 =$	8) $+5a - -2a =$
9) $-350 - -250 =$	9) $+7a + -4a =$
10) $-275 - -175 =$	10) $-5a + +3a =$

- What is it. - An exercise in exponents.
- Why use it. - Some pupils are fascinated by working with exponents. Use it to develop skill in working with powers.
- When to use it. - Use it along with the work in solving the right triangle.
- How to use it. - Ask pupils to give the simplest name for each expression in accordance with their understanding of the meaning of exponents.

Give the simplest number name for each of the following.

1. $3^3 =$

2. $3^2 =$

3. $3 \cdot 3 \cdot 3 =$

4. $3^3 =$

5. $3 \cdot 3 \cdot 3 \cdot 3 =$

6. $3^4 =$

7. $2^3 =$

8. $2^5 =$

9. $5^2 =$

10. $5^3 =$

11. $7^3 =$

12. $7^2 =$

13. $7^1 =$

14. $7^0 =$

15. $(-\frac{1}{2})^2 =$

16. $(-\frac{1}{3})^2 =$

17. $2^2 \cdot 2^3 =$

18. 5^2

19. $12^2 =$

20. $15^2 =$

BIBLIOGRAPHY OF MATHEMATICS ENRICHMENT PUBLICATIONS

1. Abbott, E. A. FLATLAND. Dover Publications, Inc., 920 Broadway, New York, New York 10016, 1952, 117 pp., paper. \$1.00

A fantasy about life in a two dimensional world. Explains certain aspects of modern science.

2. Adler, Irving. MAGIC HOUSE OF NUMBERS. John Day Company, 200 Madison Ave., New York, New York, 1957, 128 pp., cloth. \$3.00

Our number system made interesting via mathematical curiosities, riddles, tricks, and games.

3. Bakst, Aaron. MATHEMATICAL PUZZLES AND PASTIMES. D. Van Nostrand Co., Inc., 120 Alexander St., Princeton, New Jersey, 1954, 206 pp., cloth. \$4.00

Some amusing recreations that provide for fun and relaxation are brought together. Includes the curious, the strange, and the seemingly impossible.

4. Bakst, Aaron. MATHEMATICS, ITS MAGIC AND MYSTERY. D. Van Nostrand Co., Inc., 120 Alexander St., Princeton, New Jersey, 1952, 320 pp., cloth. \$7.50

A book about numbers and their relation to life. Brings alive the magic of figures and shows that mathematics is not a dry and dusty subject, but a strong and inspiring tool to solve all sorts of problems. Some complex mathematics is made reasonably clear.

5. Ball, W. W. R. MATHEMATICAL RECREATIONS AND ESSAYS. The Macmillan Co., 60 Fifth Ave., New York, New York 10011, 1960 (11th edition), 418 pp., cloth. \$3.95

Includes puzzles, problems, cryptography, card tricks, geometric constructions, and essays. Materials for a wide range of ability levels.

6. Bell, Erie T. MEN OF MATHEMATICS. Simon & Schuster, Inc., 630 Fifth Ave., New York, New York 10020, 1961, 592 pp., paper. \$2.25

The exciting lives of the world's great mathematicians. Humorous and philosophical.

7. Bell, Erie T. NUMEROLOGY. Wehman Brothers, 712 Broadway, New York, New York 10003, 1933, 187 pp., cloth. \$2.50

Some tricks and puzzles which lie on the fringes of the history of mathematics. Superstitions about numbers are explored. Clever and humorous.

8. Bell, R. C. BOARD AND TABLE GAMES FROM MANY CIVILIZATIONS. Oxford University Press, 417 Fifth Ave., New York, New York 10016, 1960, 232 pp., cloth \$5.00

Ninety-one games from many civilizations described; the oldest was played some five thousand years ago.

9. Boehm, George A. W. and the Editors of Fortune. THE NEW WORLD OF MATH. The Dial Press, Inc., 461 Park Ave., South, New York, New York 10016, 1959, cloth. \$2.50

A book about the universal language of science; some difficult concepts translated into intelligible terms.

10. Bowers, Henry and Joan. ARITHMETIC EXCURSIONS; AN ENRICHMENT OF ELEMENTARY MATHEMATICS. Dover Publications, Inc., 920 Broadway, New York, New York 10016 1961, 364 pp., paper. \$1.65

Little known arithmetical facts, diversions, puzzles, and methods. Answers, with explanations, included.

- *11. Brandes, Louis G. A COLLECTION OF CROSS-NUMBER PUZZLES. J. Weston Walch, Publisher, P. O. Box 1075, Portland, Maine, 1958, 249 pp., paper, teacher edition \$2.50; student edition \$2.00

A collection of one hundred four cross-number puzzles for classroom use; suggestions for making cross-number puzzles.

- *12. Brandes, Louis G. GEOMETRY CAN BE FUN. J. Weston Walch, Publisher, P. O. Box 1075, Portland, Maine, 1958, 249 pp., paper, teacher edition \$2.50; student edition \$2.00

A collection of supplementary mathematics material for use with general mathematics and geometry classes. Includes oddities, stories, puzzles, tests, games, projects, constructions, and problems.

- *13. Brandes, Louis G. YES, MATH CAN BE FUN! J. Weston Walch, Pub. P.O. Box 1075, Portland, Maine, 1960, 263 pp., paper, teacher edition \$2.50; student edition \$2.00

A collection of enrichment materials for use with junior high school mathematics classes. Includes number oddities, puzzles, tricks, games, stories, recreational tests, optical illusions, problems, and suggested projects.

14. Court, Nathan A. MATHEMATICS IN FUN AND ERNEST. The Dial Press, Inc., 461 Fourth Ave., New York, New York 10016, 1958, 250 pp., cloth, \$4.75

A collection of essays on various phases of mathematics; what mathematics is and what mathematicians do.

15. Cundy, H. M. and Rollett, A. P. MATHEMATICAL MODELS. Oxford-Clarendon Press, 417 Fifth Ave., New York, New York 10016, 1952, 240 pp., cloth. \$5.50

A guide for building models of mathematical curves and formulas. Provides enrichment material for advanced students.

16. Cutler, Ann and McShane, Rudolph. THE TRACHTENBERG SPEED SYSTEM OF BASIC MATHEMATICS. Doubleday & Co., Inc., Garden City, New York, 1960, 270 pp., cloth. \$4.95

Explains a system of high-speed arithmetical calculations.

17. Degrazia, Joseph. MATH IS FUN. Emerson Books, Inc., 251 W. 19th St., New York, New York 10011, 1954, 159 pp., cloth. \$2.95

A collection of number oddities and relationships, problems, and puzzles. Solutions are included. No knowledge beyond high school mathematics necessary for most of the puzzles.

18. Dudeney, H. E. AMUSEMENTS IN MATHEMATICS. Dover Publications, Inc., 920 Broadway, New York, New York 10016, 1958, 258 pp., paper. \$1.25

A collection of puzzles and problems by a famous British puzzle-maker. The puzzles and problems vary from quite simple to those that are of a high order of difficulty.

19. Fadiman, Clifton (Editor). FANTASIA MATHEMATICS. Simon & Schuster, Inc., 630 Fifth Ave., New York, New York 10020, 1961, 298 pp., paper. \$1.45

An assortment of stories and sketches to show the fun one can have with mathematics.

- *20. Freeman, Mae B. THE STORY OF ALBERT EINSTEIN. Random House, 457 Madison Ave., New York, New York 10022, 1958, 178 pp., cloth. \$2.95

A biography of the shy, dreamy boy who was always asking, "Why?" and who later became one of the greatest mathematicians of all time.

- *21. Freeman, Mae B and Ira. FUN WITH FIGURES. Random House, 457 Madison Ave., New York, New York 10022, 1946, 60 pp., cloth. \$1.95

Tells how to have fun with geometric figures and at the same time develop an understanding of their meaning. Some sixty photographs and diagrams.

22. Gamow, George. ONE, TWO, THREE: INFINITY. The Viking Press, 625 Madison Ave., New York, New York 10022, 1961, 352 pp., cloth, \$5.00, paper. \$1.65

Facts about and speculations about science written for the layman. Includes natural and artificial numbers, space and time concepts, the fourth dimension, the atomic theory, and new ideas of astronomy.

23. Gamow, George and Stern, Marvin. PUZZLE-MATH. The Viking Press, Inc., 625 Madison Ave., New York, New York 10022, 1958, 128 pp., cloth. \$2.75

Brain-twisters and puzzles based on everyday situations that can be untangled by mathematical thinking.

24. Gardner, Martin. THE SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES AND DIVERSIONS. Simon & Schuster, Inc., 630 Fifth Ave., New York, New York 10020, 1959, 178 pp., cloth. \$3.50

A collection of popular mathematical games from SCIENTIFIC AMERICAN MAGAZINE. Includes paper folding, paradoxes, magic squares, topological curiosities, and problems.

25. Graham, L. A. INGENIOUS MATHEMATICAL PROBLEMS AND METHODS. Dover Publications, Inc., 920 Broadway, New York, New York 10016, 1959, 237 pp., paper. \$1.45

A collection of one hundred puzzles contributed by mathematicians to a magazine over a period of time.

26. Heafford, Philip. THE MATH ENTERTAINER. Emerson Books, Inc., 251 West 19th St., New York, New York 10011, 1959, 176 pp., cloth. \$2.95

Subject matter ranges through mathematical history, symbols, circles, conic sections, units, measures, series, permutations, abbreviations, and a variety of posers. Includes answers and explanations.

- *27. Heath, R. V. MATHEMAGIC. Dover Publications, Inc., 920 Broadway, New York, New York 10016, 1953, 138 pp., cloth. \$1.00

A Stunt book. Includes a wide variety of tricks, games, and puzzles. Shows how to multiply large numbers rapidly and figure interest rates quickly. Prepared for high school students.

- *28. Hogben, L. T. THE WONDERFUL WORLD OF MATHEMATICS. Doubleday & Co., Inc., Garden City, New York, 1955, 72 pp., cloth. \$2.95

The story of how man discovered mathematics to help him sow crops, navigate, and build cities. Man learned to count, measure land, and measure distance to the planets. Many illustrations.

29. Hogben, L. T. MATHEMATICS FOR THE MILLIONS. W. W. Norton & Co., Inc., 101 Fifth Ave., New York, New York 10003, 1951, 697 pp., cloth. \$6.95

Stresses historical and social aspects of mathematics. Shows application of the branches of mathematics to life problems.

30. Huff, Darrell and Geis, Irving. HOW TO TAKE A CHANCE. W. W. Norton and Co., Inc., 101 Fifth Ave., New York, New York 10003, 1959, 142 pp., cloth. \$2.95

Humorous, but authoritative. An explanation of the laws of chance. Cartoon illustrated.

31. Huff, Darrell and Geis, Irving. HOW TO LIE WITH STATISTICS. W. W. Norton and Co., Inc., 101 Fifth Ave., New York, New York 10003, 1954, 142 pp., cloth. \$2.95

Humorous, but authoritative. An explanation of the right and wrong uses of statistics. Cartoon illustrated.

32. Hunter, J. A. FIGURETS: MORE FUN WITH FIGURES. Oxford University Press, 1600 Polliot Dr., Fair Lawn, New Jersey, 1958, 128 pp., cloth. \$3.50

Interesting problems cast in the form of entertaining anecdotes.

33. Hunter, J. A. FUN WITH FIGURES. Oxford University Press, 1600 Polkit Dr., Fair Lawn, New Jersey, 1956, 160 pp., cloth. \$3.00

A collection of arithmetical puzzles.

34. Jacoby, Oswald. HOW TO FIGURE THE ODDS. Doubleday & Co., Inc., Garden City, New York, 1947, 215 pp., cloth. \$2.95

Cleverly written book on the odds of chance by an authority on card games. Gives the odds on all sorts of things. Provides excellent material for an introduction to the subject of probability.

35. James, Robert C. and Glen. MATHEMATICS DICTIONARY. D. Van Nostrand Co., Inc., 120 Alexander St., Princeton, New Jersey, 546 pp., cloth. \$15.00

Explicit definitions of mathematical terms, concepts, and relationships; multi-lingual indexes.

- *36. Johnson, Donovan A. PAPER FOLDING FOR MATHEMATICS CLASSES. National Council of Teachers of Mathematics, 1201 Sixteenth St., N. W., Washington, D. C., 1957, 36 pp., paper. \$.75

A collection of paper folding exercises for use with mathematics classes.

37. Jones, B. W. ELEMENTARY CONCEPTS OF MATHEMATICS. The Macmillan Co., 60 Fifth Ave., New York, New York 10011, 1947, 294 pp., cloth. \$5.50

Includes puzzle materials, non-scientific mathematical applications such as topology and mirror geometry, probability, and other topics.

- *38. Jones, S. E. MATHEMATICAL CLUBS AND RECREATIONS. S. I. Jones Co., 1122 Belvidere Dr., Nashville, Tennessee, 1940, 236 pp., cloth. \$3.00

A thorough discussion of mathematics clubs; a selection of recreations is included. Has many suggestions for organizing and conducting mathematics clubs. Recreations include problems, number oddities, number rhymes, games, puzzles, and riddles. A must for the mathematics club sponsor.

39. Jones, S. I. MATHEMATICAL NUTS. S. I. Jones Co., 1122 Belvidere Dr., Nashville, Tennessee, 1936, 340 pp., cloth. \$3.50

A book of recreational problems from arithmetic to calculus with their solutions. Divided into sections for various interest and ability levels.

40. Jones, S. I. MATHEMATICAL WRINKLES. S. I. Jones Co., 1122 Belvidere Dr., Nashville, Tennessee, 1929, 361 pp., cloth. \$3.50

A handbook for both teachers and students on recreational mathematics. Covers many of the areas that can be considered under the heading of recreational mathematics, including problems, puzzles, paradoxes, probabilities, topology, geometric relations, and special topics.

41. Kasner, Edward and Newman, James. MATHEMATICS AND THE IMAGINATION. Simon & Schuster, Inc., 630 Fifth Ave., New York, New York 10020, 1940, 380 pp., cloth. \$4.50

A guide tour through the world of post-Einsteinian mathematics. The authors reveal some of the tricks and fascinations of higher mathematics.

42. Kaufman, G. L. THE BOOK OF MODERN PUZZLES. Dover Publications, Inc., 980 Broadway, New York, New York 10016, 1954, 192 pp., paper. \$1.00

A book of various kinds of puzzles. Contents include simple two-minute teasers, involved word labyrinths, design and pattern puzzles and others.

43. Larson, Harold D. ENRICHMENT PROGRAM FOR ARITHMETIC. Harper & Row Publishing Co., Evanston, Illinois, 1960, paper. \$1.60

Eight 16-page books for each of grades three through eight.

44. Lee, W. W. MATH MIRACLES. W. W. Lee, Box 105, Durham, North Carolina, 1960, 76 pp., cloth. \$3.00

Some mathematical magic, written by and for magicians. Clearly described and illustrated.

45. Leeming, Joseph. MORE FUN WITH PUZZLES. J. B. Lippincott Co., East Washington Square, Philadelphia, Pennsylvania 19105, 1947, 149 pp., cloth. \$2.95

Entertaining mathematical and number puzzles; also match, coin and counter puzzles.

46. Leeming, Joseph. FUN WITH PUZZLES. J. B. Lippincott Co., East Washington Square, Philadelphia, Pennsylvania 19105, 1946, 128 pp., cloth. \$3.50

A collection of many kinds of puzzles, including coin and match puzzles, cut-out puzzles and others; includes answers.

47. Lieber, L. R. THE EDUCATION OF T. C. MITTS. W. W. Norton & Co., Inc., 101 Fifth Ave., New York, New York 10003, 1944, 229 pp., cloth. \$3.95

An exposition of classical mathematics for the celebrated man in the street, plus a treatment of the new mathematics, including life in a four dimensional world. A number of similar books by this author if this one appeals.

48. Merrill, Helen A. MATHEMATICAL EXCURSIONS. Dover Publications, Inc., 980 Broadway, New York, New York 10016, 1950, 145 pp., paper. \$1.00

A collection of short cuts, magic squares, and recreational problems. Prepared for the high school students.

49. Meyer, J. S. FUN WITH MATHEMATICS. Harcourt, Brace & World, Inc., 7555 Caldwell Ave., Chicago, Illinois 60648, 1952, 176 pp., cloth. \$2.95 also Fawcett World Library, 67 W. 44th St., New York, New York 10036, paper. \$.50

Mathematical puzzles, formulas, magic, and tricks that provide fun and information. Intended for young people.

50. Mott-Smith, Godfrey. MATHEMATICAL PUZZLES FOR BEGINNERS AND ENTHUSIASTS. Dover Publications, Inc., 980 Broadway, New York, New York 10016, 1954, 248 pp., paper. \$1.00

A collection of mathematical puzzles and their solutions. For beginners, as well as the trained mathematicians.

51. Newman, James R. (Editor) THE WORLD OF MATHEMATICS (four vol., boxed) Simon & Schuster, Inc., 630 Fifth Ave., New York, New York 10020, 1956, more than 2500 pp., cloth. \$25.00, paper. \$9.95

An extensive collection of literature of mathematics, including its history, philosophy, and bearing on science; a mathematics library itself.

52. Northrop, E. P. RIDDLES IN MATHEMATICS: A BOOK OF PARADOXES. D. Van Nostrand Co., Inc., 120 Alexander St., Princeton, New Jersey, 1944, 270 pp., cloth. \$4.50

More than two hundred riddles and paradoxes drawn from every branch of mathematics. Provides a wealth of material on fundamental reasoning.

53. Pedoe, Dan. THE GENTLE ART OF MATHEMATICS. The Macmillan Co., 60 Fifth Ave., New York, New York 10011, 1959, 143 pp., cloth. \$3.95

The world of numbers presented in an amusing fashion. Includes mathematical games, laws of chance, automatic thinking, symmetry and series, and rational and irrational numbers. Prepared for those wanting to know more about modern mathematics.

54. Rademacher, Jans and Toeplitz, Otto. THE ENJOYMENT OF MATHEMATICS. Princeton University Press, Princeton, New Jersey, 1957, 240 pp., cloth. \$4.50

Introduces some basic ideas of mathematics without requiring more than a background of high school algebra and geometry. Recreational, aesthetic, and instructive. Supplies supplementary material for teachers.

55. Ranson, William R. ALGEBRA CAN BE FUN! J. Weston Walch, Publisher, P.O. Box 1075, Portland Maine, 1958, 195 pp., paper. \$2.50

Algebra curiosities and unusual problems, with solutions; most at second year algebra level of achievement.

56. Ranson, Willian R. ONE HUNDRED MATHEMATICAL CURIOSITIES. J. Weston Walch, Publisher, P. O. Box 1075, Portland, Maine, 1955, 212 pp., paper. \$3.00

A simplified presentation of some advanced material that includes problems, geometrical constructions, and facts of an enrichment nature.

- *57. Ravielli, Anthony. AN ADVENTURE IN GEOMETRY. The Viking Press, Inc., 625 Madison Ave., New York, New York 10022, 1957, 177 pp., cloth. \$3.00

Geometric forms from the world in which we live are beautifully illustrated and described. Provides for an appreciation of the beauty that is mathematics.

58. Reichmann, W. J. THE FASCINATION OF NUMBERS. Oxfird University Press, 1600 Pollit Dr., Fair Lawn, New Jersey, 1957, 176 pp., cloth. \$4.00

A presentation of various number relationships, including magic squares, number peculiarities, pseudo-telepathy, and other mathematical recreations.

59. Ringerberg, Lawrence A. A PORTRAIT OF 2. National Council of Teachers of Mathematics, 1201 Sixteenth St., N. W., Washington, D. C., 1956, 48 pp., paper. \$.75

Discusses 2 as an integer, a rational number, a real number and a complex number. Lays a foundation for modern mathematics.

60. Schaaf, W. L. RECREATIONAL MATHEMATICS. National Council of Teachers of Mathematics, 1201 Sixteenth St., N. W., Washington, D. C., 1958, 151 pp., paper. \$1.20

A bibliography listing materials in fifth-six categories under various headings.

61. Smeltzer, Donald. MAN AND NUMBERS. Emerson Books, Inc., 251 W. 19th St., New York, New York 10011, 1957, 114 pp., cloth. \$2.50

An account of man's use of numbers through the ages; from learning to count through the modern number system.

62. Steinhaus, Hugo. MATHEMATICAL SNAPSHOTS. Oxford University Press, 1600 Pollit Dr., Fair Lawn, New Jersey, 1960, 336 pp., cloth. \$6.75

The purpose of the book is to visualize mathematics. Many mathematical diagrams and photographs. Of special interest to plane and solid geometry students.

63. Tocquet, Robert. MAGIC OF NUMBERS. Wehman Brothers, 712 Broadway, New York, New York 10003, 1960, 160 pp., cloth. \$3.50

Mental calculations, tricks of memory, trick mathematics, calculating prodigies.

64. White, W. F. A SCRAPBOOK OF ELEMENTARY MATHEMATICS. The Open Court Publishing Co., LaSalle, Illinois, 1908, 248 pp., cloth. \$3.00

A collection of accounts, essays, recreations and notes selected for their value in arousing the interest of students in mathematics. An older publication, but a good one.

65. Williams, Eugenia. AN INVITATION TO CRYPTOGRAMS. Simon & Schuster, Inc. 630 Fifth Ave., New York, New York 10020, 1959, cloth. \$2.95

Describes the approaches and tricks having to do with cryptology. One hundred fifty puzzles, with solutions; range from simple to moderately difficult.

66. Wylie, C. R. 101 PUZZLES IN THOUGHT AND LOGIC. Dover Publications, Inc., 920 Broadway, New York, New York 10016, 1958, 115 pp., paper. \$1.00

Puzzles of purely logical nature.

* Believed to be of special interest to junior high students.

Robert C. Madison
Russell D. Ackley

A FEW REFERENCES:

1. Spitzer, Herbert F. PRACTICAL CLASSROOM PROCEDURES FOR ENRICHING ARITHMETIC. Webster Division, McGraw-Hill Book Co.

This is a collection of materials for enrichment of the teaching of arithmetic to be used in grades 4-8. Includes games, puzzles, number tricks, new approaches to the basic number skills, and exercises in construction and arrangement.

2. Spitzer, Herbert F. ENRICHMENT ACTIVITIES FOR GRADE 8. McGraw-Hill Book Co.

Contains a collection of unusual number questions and problems, also arithmetical puzzles, tricks, construction exercises, and so on.

3. Irving and Eastman. HIGHWAY TO MATH FUN. Harr Wagner Publishing Co.

A collection of Dot-to-Dot, Add-a-Trail, Math-O-Squares, and Crossmath Activities.

4. Wylie, C. R., Jr. 101 PUZZLES IN THOUGHT AND LOGIC. Dover Publishers, Inc.

It is exactly what the title implies.

Alta B. Cameron

867

HELPFUL REFERENCE MATERIALS

1. Bernstein and Wells. TROUBLE SHOOTING MATHEMATIC SKILLS. Holt, Rinehart & Winston, Inc.

Has one chapter Mathematics and Your Work which has many good illustrations presented. Samples of how worker's wages are figured including overtime; explanation of "withholding" taxes and other deductions which might be subtracted from workers checks; and samples of stubs and checks used to pay workers.

2. Spitzer. PRACTICAL CLASSROOM PROCEDURES FOR ENRICHING ARITHMETIC. Webster Division, McGraw-Hill Book Co.

Has very simple games, puzzles, and tricks.

3. Spitzer. ACTIVITIES FOR THE ENRICHMENT OF ARITHMETIC PART 7. McGraw-Hill Book Co.

Includes exploration and discovery activities, games, tricks, and recreational activities.

4. Gundlach. THE LAIDLAW GLOSSARY OF ARITHMETICAL-MATHEMATICAL TERMS. Laidlaw Brothers, Publishers

Elsie B. Sawyers

BOOKS FOR MATHEMATICS

Meyer, Jerome S. FUN WITH MATHEMATICS, Harcourt, Brace & World, Inc., 176 pp., cloth \$2.95

From the wonders of the Nomograph, which solves quadratic equations to a section on magic squares, this book provides a wide assortment of mathematical puzzles. All kinds of mathematical sleights of hand, magic squares, formulas for telling the height of a tree, house, etc. without any figuring. Baffling tricks of all kinds that are hard to figure out but easy to do. How the Romans multiplied and divided.

Adler, Irving. LOGIC FOR BEGINNERS THRU GAMES, JOKES, AND PUZZLES. The John Day Co., 160 pp. cloth \$3.95

Clear thinking and logical reasoning taught through an amusing series of games. Fun with a serious purpose, for grades 6 and up. Both entertaining and instructive and the material is logically presented.

Gamow, George and Stern, Marvin. PUZZLE-MATH. The Viking Press, Inc. 1958, 119 pp. cloth \$3.00

The authors present their brain-twisters not as mere puzzles to be solved--though the invertebrate puzzler may do so if he likes--but as stories of human situations in which a seemingly baffling problem is untangled by mathematical or logical thinking. The answers are unfolded along with the story, and the reader's enjoyment, as in a good whodunit, comes from watching the authors apply logic to a set of given facts. The authors' whimsical story telling knack and their very humane approach to higher mathematics will delight all.

Greenblatt, M. H. MATHEMATICAL ENTERTAINMENTS. 160 pp. cloth \$4.95

The mathematical entertainment in this book will provide hours of pleasure for those who revel in puzzles and brilliant solutions, demanding of the reader only a sharp pencil and a sharper wit. On every page the beauty and elegance of mathematics is superbly conveyed. There are coin-weighing problems, partitioning problems, problems involving weights, poker chips, and piles of bricks. None requires laborious calculations. Most of the problems can be done mentally, if they can be done at all. More than 30 diagrams and illustrations are included.

Degrazia, Joseph. MATH IS FUN. Emerson Books, Inc., 1954, 159 pp. cloth \$2.95

Here is a treasury of entertainment for the mathematically minded, consisting of intriguing and brain-teasing problems and puzzles. No knowledge beyond simple high school mathematics is required to solve most of the puzzles, and many call for only ordinary arithmetic. The reasoning required, however, is of a high order, some of the apparently casual problems calling for uncommon ingenuity. Solutions are given where needed.

Heafford, Phillip. THE MATH ENTERTAINER. Emerson Books, Inc., 1959, 176 pp.
cloth \$2.95

The mathematical teasers, ticklers, twisters, traps, and tricks here dangled before the reader are designed merely to puzzle, titillate, and delight. The subject matter ranges through mathematical history, symbols, circles, triangles, conic sections, units, measures, moneys, series, permutations, abbreviations, etc. Some of the posers will be easy, others decidedly less so, some will amuse, others exasperate--but none are dull. When he is all through, the reader will have a lot of fun and will have a better grasp of mathematics and mathematical reasoning.

Gardner, Martin. THE SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES & DIVERSIONS. Simon & Schuster, Inc., 1959, 178 pp. cloth \$3.50

Paradoxes and paperfolding moebius variations and mnemonics, fallacies, brain-teasers, magic squares, topological curiosities, probability and parlor tricks, and a variety of ancient and new games and problems. All with mathematical commentaries by Mr. Gardner, who is a regular contributor to Scientific American. Many diagrams. References for further reading.

Brandes, Louis G. A COLLETTION OF CROSS-NUMBER PUZZLES. J. Weston Walch, Publisher, 1958, 249 pp. Teacher's Edition \$2.50

For Grades 7-9.

Johnson. COMMERCIAL GAMES FOR THE ARITHMETIC CLASS. (Reprint from the Arithmetic Teacher, March 1, 1958) \$.20

For Grades 4-6.

Mary June Meeks

867

EXERCISES FOR EXPERTS

PURPOSE

To provide the fairly capable student with a challenge and the slower student with a realistic approach. To stress logical reasoning and skills rather than skills alone.

METHOD

Label each problem, Exercise for Experts, thus formulating the idea that by being allowed to do the exercises one is an expert in his own way. They may be presented one each day or in a group. The exercise that requires addition and the one dealing mostly with rational numbers would follow the unit on rational numbers and so on.

Exercises for Experts

There is no larger number.
The last one can't be found.
Just add me to get the next.
There is no upper bound.
Answer: One

I'm a little bitty number.
So very small indeed.
If you add me to another,
A change you cannot read.
Answer: Zero

I'm not so very big.
Just over 33.
Add some fives together,
My name you soon will see.
Answer: 35

Increase me by 5
Then take away 7.
When you are done,
You should have 11.
Answer: 13

Now I'm the only digit
Whose number stays the same
No matter where I am
In our place value game.
WHO AM I?
Answer: 0

Number thirty-one
Is slightly more than me.
I'm that much larger
Than number twenty-three.
Answer: 27

If you add me to myself
Then multiply by two,
It's 12 plus four you're sure to get.
You need no other clue.
Answer: 4

Multiply me by myself,
Or find my sum with two.
Your answer is the same
No matter which you do.
Answer: 2

My head and feet are just alike.
My terms, as low as they can be.
As a fraction, I'm not proper.
I'm one you seldom see.
Answer: $\frac{1}{1}$

Multiply me by myself
Then multiply by 2.
5000 is the answer.
You need no other clue.
WHO AM I?
Answer: 50

I'm the smallest number that
Will get you less than fifty
If you want to take away
From seven more than sixty.
WHO AM I?

Answer: 18

I'm a special number
As special as can be.
Use me as a factor
But don't divide by me.
WHO AM I?

Answer: 0

To multiply by me,
Here's what you can do.
First multiply by 10,
Then multiply by 2.
WHO AM I?

Answer: 20

I'm the smallest number
Ever to make this claim.
I have 4 prime factors,
With none of them the same.
WHO AM I?

Answer: 210

If I were four less,
Then I'd be four more
Than four hundred three
Plus two hundred four.
WHO AM I?

Answer: 615

Farmer Jones had some animals.
 $\frac{1}{4}$ were horses, $\frac{1}{2}$ were cows.
The rest were pigs.
He had 8 pigs.
How many horses and how many cows did
he have?

Answer: 8 horses, 16 cows

VARIATIONS

Contest for entire class to see who can get the right answer first.
Contest for girls only.
Contest for boys only.
Section class into various groups to see who can be right first.
Care should be taken to see that even the slowest pupil will be first
part of the time.
Allow winners to explain their method to the rest of the class.
Have pupils make up an exercise of their own and try to stump the rest
of the class. This should be done during class to prevent help from older
brothers and sisters.
Post on a bulletin board the exercises developed by pupils. This will
encourage all to see their name in lights.

REFERENCE

Richolz, O'Daffer, Brumfiel, & Shanks. BASIC MODERN MATHEMATICS, FIRST
COURSE. Addison-Wesley Publishing Co., Inc., Reading, Mass. 01867.

Russell D. Ackley

867

ROAD MAPS

PURPOSE

The gimmick used here was in the use of highway road maps. This was used in a low ability 7th grade class, and used to promote both math and social studies ideas.

In math, it helped the student develop manipulative skills, promote ideas of distance, and the ability to estimate distance.

METHOD

Each student had the same kind of map, ordered from the Iowa State Highway Commission, so that all contained the same mileage and facts. Each student was given sample problems to work. For example: How many miles is it from Denison to Ft. Dodge by way of Highway 59 and 20.

From this point each student made some of his own problems for exchange in class. Other activities included certain oral activities where the students followed certain routes and kept adding mentally as they went.

These activities provided enthusiasm and interest for the students, and helped develop the ability to follow directions and to work with more accuracy.

VARIATION

Maps of various kinds may be used as well as maps depicting larger or smaller areas.

REFERENCES

An original idea.

Charles W. Breeding

867

LUCKY NUMBERS

PURPOSE

An exercise for the student to help them get used to following directions and at the same time have them develop a meaningful subset of the Natural numbers. Also, it offers a chance to check the student's understanding of prime numbers.

METHOD

To find Lucky Numbers, make a sieve of natural numbers. Take the first number after one. It is 2. Cross out every second numeral (starting with 2). Look at the numerals left. Take the first one after 1. It is 3. Cross out every third numeral (starting with 5). (Never cross out a numeral twice.)

Look at the numerals left. Take the first one after 1. It is 3. We have already used it, so take the next one. It is 7. Cross out every 7th numeral of those that are left.

Continue until all possible numerals are crossed out. The first few Lucky numerals are 1, 3, 7, 9, 13, 15, 21, 25, 31.

How are Lucky numbers like prime numbers?

How are they different?

What can you discover about Lucky numbers?

Answer.

Like -- There are Lucky number twins.

Unlike -- All Lucky numbers are odd numbers.

Ira Burchett

8-67

MATH QUICKIES

PURPOSE

These are short math 'quickies' to be used as a motivational aid. They may be used the last few minutes of class or as a homework problem.

METHOD

Either have the problem on a ditto sheet so that each student may receive a copy or write the problem on the board.

VARIATION

Place one such problem on the bulletin board each Friday and let the students who wish to work on it do so over the weekend.

Some children are seated at a large round table. They pass around a box of candy. Joseph takes the first piece. Each child in turn takes a piece until all 50 pieces are gone. Joseph also got the last piece (and maybe others). What can you tell about the number of children at the table?

Answer: There were 7 or 49 children.

The Drury family is made up of 1 grandmother, 1 grandfather, 2 mothers, 2 fathers, 4 children, 3 grandchildren, 1 brother, 2 sisters, 2 sons, 2 daughters, 1 mother-in-law, 1 father-in-law, and 1 daughter-in-law. How many people are there in the family?

Answer: Seven

The Andersons were about to start on an 18,000 mile automobile trip. They had their tires checked and found that each one was good for 12,000 miles and not one bit more. How many spares will they need to make the trip?

Answer: 2 -- Change two tires at 6000 miles.

Ira Burchett

867

PATTERNS

The following items were contributed by Genevieve Drey, a teacher at Irving Junior High School, Des Moines, Iowa.

A. Try it.

1. Choose any number.
2. Multiply it by 6.
3. Add 12.
4. Divide by 2.
5. Subtract 6.
6. Divide by 3.

Can you prove why this works?

B. The magic number, 1089.

1. Write a three digit number such that the first digit is one more than the second, and the second is one more than the third.
2. Write a number by reversing these digits.
3. Subtract the second number from the first.
4. Reverse the digits of this answer.
5. Add this new number to the answer.

C. Age puzzle.

1. Write your month of birth as a numeral.
2. Multiply this by 2.
3. Add 5.
4. Multiply by 50.
5. Add your present age.
6. Subtract 250.

What does the last two digits tell you? The first digit or digits?

D. Squares (Two-dimensional)

$$1=1$$

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

and so on.

E. Cubes (Three-dimensional)

$$1 = 1$$

$$3 + 5 = 8$$

$$7 + 9 + 11 = 27$$

$$13 + 15 + 17 + 19 = 64$$

and so on.

Patterns (continued)

F. See D.

$$1^2 = 1$$

$$2^2 = (1+2) + 1$$

$$3^2 = (1+2+3) + (1+2)$$

$$4^2 = (1+2+3+4) + (1+2+3)$$

and so on.

G. Triangles

(Triangular numbers, as compared with square and cube numbers.)

0

oo

ooo

1

3

6

1

1 + 2

1 + 2 + 3

and so on.

H. Using only paper and pencil, what is the quickest way to add the numbers 1 through 100?

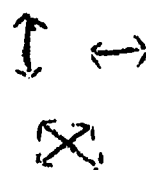
Karl Gauss, as a German schoolboy, did this as follows:

$$\frac{100 (1 + 100)}{2} \quad \text{Why does this work?}$$

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

I. Just the Digits

8	1	6
3	5	7
4	9	2



Magic squares are over 3000 years old. The one above appeared in the year 1514.

PATTERNS

J. Durer's Puzzle -- 1514 A.D.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Can you get 34, thirty-four different ways? Some are ↓ ↓ ↓ ↓
→ → → → ✕ corners,
four quarters, center block,
etc.



L. Upside - down Magic Square.

96	11	89	68
88	69	91	16
61	86	18	99
19	98	66	81

They add up to 264. Turn it upside down. There are about 48 ways to get 264.

K. Another to Try.

12	2	1	15
7	9	10	4
11	5	6	8
0	14	13	3

See how many different paths will lead to 30.

The next two are unusual. Junior high pupils always enjoy them.

M. 1 x O H O x 1

8818	1111	8188	1881
8181	1888	8811	1118
1188	8881	1818	8111

They add up to 19,998 horizontally. Now hold it upside down. Now hold it to a mirror. Now hold it to a mirror in an upside-down position.

PATTERNS

Pepping Up Multiplication Practice

A. $1 \times 8 + 1 = 9$

$$12 \times 8 + 2 = 98$$

$$123 \times 8 + 3 = 987$$

and so on.

B. $1 \times 9 + 2 = 11$

$$12 \times 9 + 3 = 111$$

$$123 \times 9 + 4 = 1111$$

and so on.

C. $3 \times 37 = 111$

$$6 \times 37 = 222$$

$$9 \times 37 = 333$$

$$12 \times 37 = 444$$

and so on.

D. Try using multiples of 3, as above, with 33, 36 and 39, instead of 37.

E. $1 \times 9 = 10 - 1$

$$2 \times 9 = 20 - 2$$

$$3 \times 9 = 30 - 3$$

and so on.

F. $1 \times 8 = 10 - 2$

$$2 \times 8 = 20 - 4$$

$$3 \times 8 = 30 - 6$$

and so on.

G. Squares

1. Select four consecutive integers.
2. Multiply them together.
3. Add one.
4. What two factors could you have used to obtain this product?

PATTERNS AND PUZZLES

PURPOSE

For interest.

METHOD

I would probably write the question on the board some morning and not mention it to any class. I am assuming that the students would discover it on the board and discuss it among themselves. After two or three days, I would have the answer underneath. I probably would not discuss these at all unless the student wished to. I would change the question at least once a week.

VARIATIONS

Could present differently.

REFERENCES

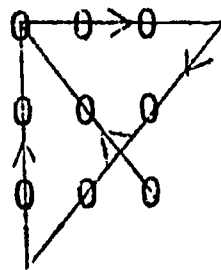
Collected from my sons who were exposed to them in various math courses.

A farmer planted ten trees in three rows with four trees in each row.
How did he plant them?

```
      X  X
     X  X  X  X
      X  X
      X  X
```

Without lifting the pencil from the paper, draw four line segments
passing through the centers of the nine circles.

```
  0  0  0
  0  0  0
  0  0  0
```



THE KING WANTED TO GET RID OF ONE OF HIS ADVISERS. HE TOOK A HAT AND PUT TWO SLIPS OF PAPER IN IT. HE TOLD EVERYONE THAT ONE WAS MARKED "LIFE" AND ONE "DEATH." HE THEN HIRED A COMPLETELY FAIR JUDGE TO DECIDE. THE KING THEN MARKED BOTH SLIPS "DEATH" BUT WHEN THE CRAFTY OLD ADVISER SHOWED A SLIP TO THE JUDGE, THE JUDGE DECIDED IN HIS FAVOR. WHAT DID THE ADVISER DO?

The adviser drew a slip and promptly tore it into pieces. He told the judge that was his choice. He then showed the remaining slip to the judge who read death and decided the adviser's choice must have been life.

A MAN WITHOUT EYES SAW APPLES ON A TREE
HE NEITHER TOOK APPLES NOT LEFT APPLES
HOW COULD THIS BE?

A man with one eye saw 2 apples on a tree and he took 1 of them.

Martha Wilkins

867

TRIANGLES

PURPOSE

Primarily for interest. It also gives the student practice in naming triangles.

METHOD

Given to the student when triangles and the naming of triangles are studied. I collect and correct only those on which the student thinks he has more than 40 triangles correctly named.

VARIATIONS

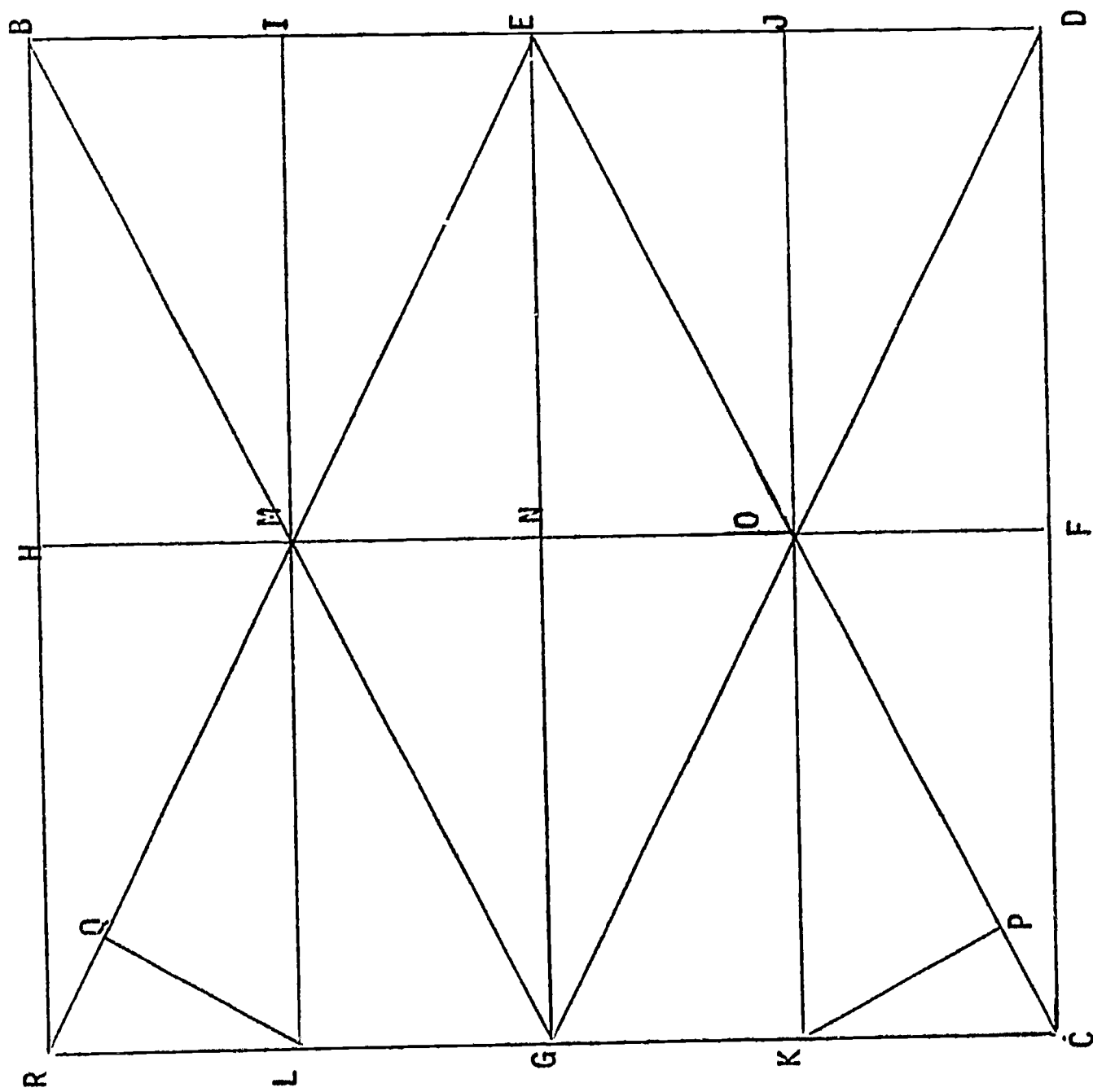
Could also be used as practice in finding the right triangle, isosceles triangle, and equilateral triangle.

REFERENCES

Taken from my son's activity book (long gone).

Martha Wilkins

867



PICTURE TRIANGLES

PURPOSE

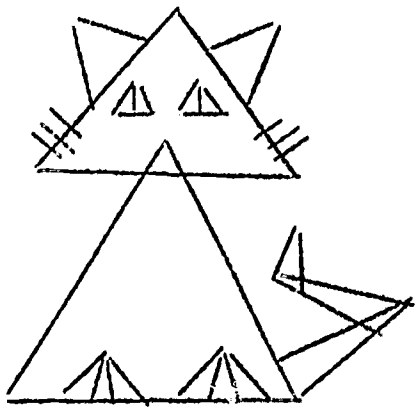
To interest the students in recognizing triangles of different shapes and sizes.

METHOD

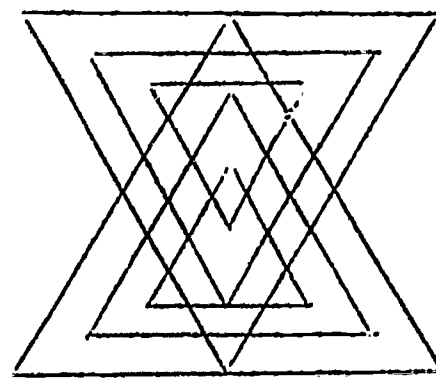
Draw the pictures on separate pieces of light colored construction paper. Place each on a small bulletin board; or in a conspicuous place where the students can stop to look at it.

VARIATION

When the study of geometric shapes is begun. Ask students to make other pictures containing other geometric shapes.



Answer: 30 triangles



Answer: 75 triangles

Alta B. Cameron

867

MATH CROSS NUMBER PUZZLES

PURPOSE

The following four pages are recreational math designed to create interest and stimulate learning. Good for using near the end of the period when the students are through with their assignments.

METHOD

On the first two pages insert numbers under 10 in the empty spaces such as to provide the correct totals.

On the last two pages insert the correct operational signs in the empty spaces such as to provide the indicated results.

VARIATIONS

1. With the slow groups draw roughly on the board and work as a group.
2. Average ability and above -- work as individuals from ditto sheets. Work in pairs competing against each other.

REFERENCE

Brandes, Louis, MATH CAN BE FUN, J. Weston Walch, Publisher, Portland, Maine

3	x		÷		=	6
+	X	+	X	x	X	-
	÷		+		=	
-	X	÷	X	÷	X	+
	+		÷		=	6
=	X	=	X	=	X	=
2	-	2	+		=	6

9	÷		+		=	8
-	X	+	X	-	X	÷
	+		-		=	
÷	X	÷	X	÷	X	+
	+		÷		=	
=	X	=	X	=	X	=
2	x		-	2	=	4

5	x		-		=	6
+	X	+	X	+	X	+
	÷		+		=	3
÷	X	-	X	÷	X	-
	x		-		=	
=	X	=	X	=	X	=
3	+		÷	2	=	2

3	x		÷		=	6
+	X	x	X	+	X	-
	+		-		=	4
+	X	÷	X	-	X	÷
	+		÷		=	
=	X	=	X	=	X	=
12	÷		-	3	=	1

Insert numbers under 10 in empty spaces such as to provide the correct totals.

(CORRECTED)

7	×		÷		=	14
+	×	×	×	-	×	-
	+		÷		=	7
×	×	÷	×	+	×	×
	×		÷		=	
=	×	=	×	=	×	=
36	÷	12	+	11	=	14

1	×		÷		=	3
×	×	×	×	×	×	+
	-		+		=	13
-	×	÷	×	÷	×	+
	+		÷		=	
=	×	=	×	=	×	=
4	+		+	8	=	20

2	+		-		=	
×	×	÷	×	×	×	+
	×		÷		=	16
-	×	+	×	-	×	-
	+		-		=	10
=	×	=	×	=	×	=
8	÷		+	11	=	12

4	+		÷		=	
×	×	+	×	×	×	+
	+		-		=	6
+	×	÷	×	÷	×	+
	×		+		=	14
=	×	=	×	=	×	=
14	+		+	3	=	22

Insert numbers under 10 in the empty spaces such as to provide the correct totals.

1		6		2	=	3
	X		X		X	
9		4		8	=	5
	X		X		X	
5		3		2	=	4
=	X	=	X	=	X	=
2		8		5	=	11

2		8		4	=	4
	X		X		X	
6		8		3	=	16
	X		X		X	
4		1		7	=	11
=	X	=	X	=	X	=
3		1		5		9

9		3		7	=	10
	X		X		X	
6		2		8	=	4
	X		X		X	
4		5		5	=	4
=	X	=	X	=	X	=
12		1		3		10

5		3		9	=	6
	X		X		X	
8		2		1	=	3
	X		X		X	
3		4		2	=	10
=	X	=	X	=	X	=
4		1		5	=	8

Insert the correct operational signs in the empty spaces as to provide the indicated results.

8		7		4	=	5
	X		X		X	
9		5		2	=	2
	X		X		X	
6		3		9	=	2
=	X	=	X	=	X	=
11		9		11	=	9

2		8		4	=	4
	X		X		X	
6		8		3	=	16
	X		X		X	
8		7		3	=	4
=	X	=	X	=	X	=
4		7		4		24

6		2		3	=	9
	X		X		X	
4		1		2	=	2
	X		X		X	
6		1		2	=	3
=	X	=	X	=	X	=
18		1		8	=	10

3		4		6	=	2
	X		X		X	
2		3		4	=	10
	X		X		X	
7		6		2	=	11
=	X	=	X	=	X	=
13		6		5	=	23

Insert the correct operational signs in the empty spaces such as to provide the indicated results.

Keith W. Emmert

867

MATHEMATICAL "CROSSWORD" PUZZLES

PURPOSE

To motivate the students to work a set of problems in such a manner that the correct answers will complete the "crossword" puzzles.

METHOD

1. Pretest at the beginning of each unit.
2. Review exercises at the beginning of the school year in junior high.
3. To vary a unit in order to stimulate or maintain interest.
4. To administer before or after a holiday.
5. To provide review or seat work if the regular teacher is absent.

VARIATIONS

1. Work individually during class period.
2. Work individually as an assignment for the next day.
3. Work in groups of two or three to solve the puzzle.
4. Work orally as a class on the overhead.
5. Girls can work the vertical problems and the boys the horizontal problems.
6. If there are four rows in a class, the first and third rows work the vertical problems and the second and fourth rows the horizontal problems.

REFERENCE

A Collection of Cross-Number Puzzles, by Louis Grant Brandes, Student Edition and Teacher Edition., J. Weston Walch, Publisher, Portland, Maine, 1957.

[This booklet contains many examples of mathematical "crossword" puzzles including the four operations, fractions, decimals, percents, simple algebra, etc. The puzzles progress from the easier puzzles to the more difficult challenging puzzles.]

Miscellaneous Puzzles -- Review for junior high students

CROSS-NUMBER PUZZLES WITH SIGNS OF OPERATION

2		8		4	=	4
6		8		3	=	16
8		7		3	=	4
=		=		=		=
4		7		4		24

6		3		9	=	9
2		5		8	=	2
6		2		4	=	8
=		=		=		=
18		10		5	=	3

A COLLECTION OF CROSS-NUMBER PUZZLES

STUDENT PUZZLE

1.	2.	3.	4.		5.	6.	
	7.			8.			9.
				10.			
11.	12.		13.				14.
15.		16.			17.	18.	
				19.			20.
	21.		22.		23.		
24.							

HORIZONTAL

- Find 25% of 5,000.
- Number of pints in 4 gallons.
- Find 20% of 60.
- $158 + 2,609 + 5,496$.
- Subtract 8,619 from 13,995.
- 5×7 .
- $3 \times 6 + 3 \times 4 + 2 \times 3$.
- Number of quarts in a peck.
- $4,760 - 60 + 1,095$.
- $5 \times 15 \frac{3}{5}$.
- $1,015.5 + 2,141.8 + 3,267.7$.
- $2,910 \div 15$.
- $241 + 152 + 101$.
- $2,034 \times 15$.

VERTICAL

- 7 is $33 \frac{1}{3}\%$ of what number?
- 5% of what number is 26?
- Subtract 3×5 from $5 - 10$.
- $410 - 87$.
- Find 20% of 1,335.
- 428 is 50% of what number?
- Find 15% of $2,453 \frac{1}{3}$.
- Take 4% of 8,750.
- $3 \times 5 + 29 + 2 \times 5 + 3$.
- Divide 210 by 6.
- $13,479 - 4,384$.
- Multiply 186 by 4.
- $1,658 \times 5$.
- $57 + 24 - 45 + 18$.
- Product of $3 \frac{1}{3}$ and 3.
- Number of days from August 10, 1956 to September 20, 1956.

Abe Willems

The figure below is a magic square. It is interesting because you get the same total if you add down each of the five columns, across the rows, or along either of the two diagonals.

11	10	4	23	17
18	12	6	5	24
25	19	13	7	1
2	21	20	14	8
9	3	22	16	15

See if you can make a magic square in the three-by-three figure below after studying the above example.

IN, INSIDE, OR OUTSIDE?

PURPOSE

To help visualize the three distinct subsets of any simple polygon in a plane, namely the simple polygon itself and its interior and its exterior.

METHOD

1. Either provide a copy for everyone or a drawing of the simple polygon on the chalkboard.
2. Ask students to make a list of the points given in the plane that are in, inside, or outside the simple polygon.
3. Try to lead the students in arriving at a rule for determining.
4. General rule to follow for determining -- from a point draw a straight line, in any direction, to the outside of the figure, then counting the number of lines this straight line intersects determines whether the point is inside or outside. If the number of lines intersected is odd then the point is inside the figure, if even number of intersections then the point is outside the figure. To determine those points in the figure is obvious as long as we agree that a point in a figure is a subset of the figure itself.
5. This idea can be done with any closed simple polygon.

VARIATIONS

This could be adapted to about any level where the teaching of polygons is done. For lower levels a more simplified figure is used.

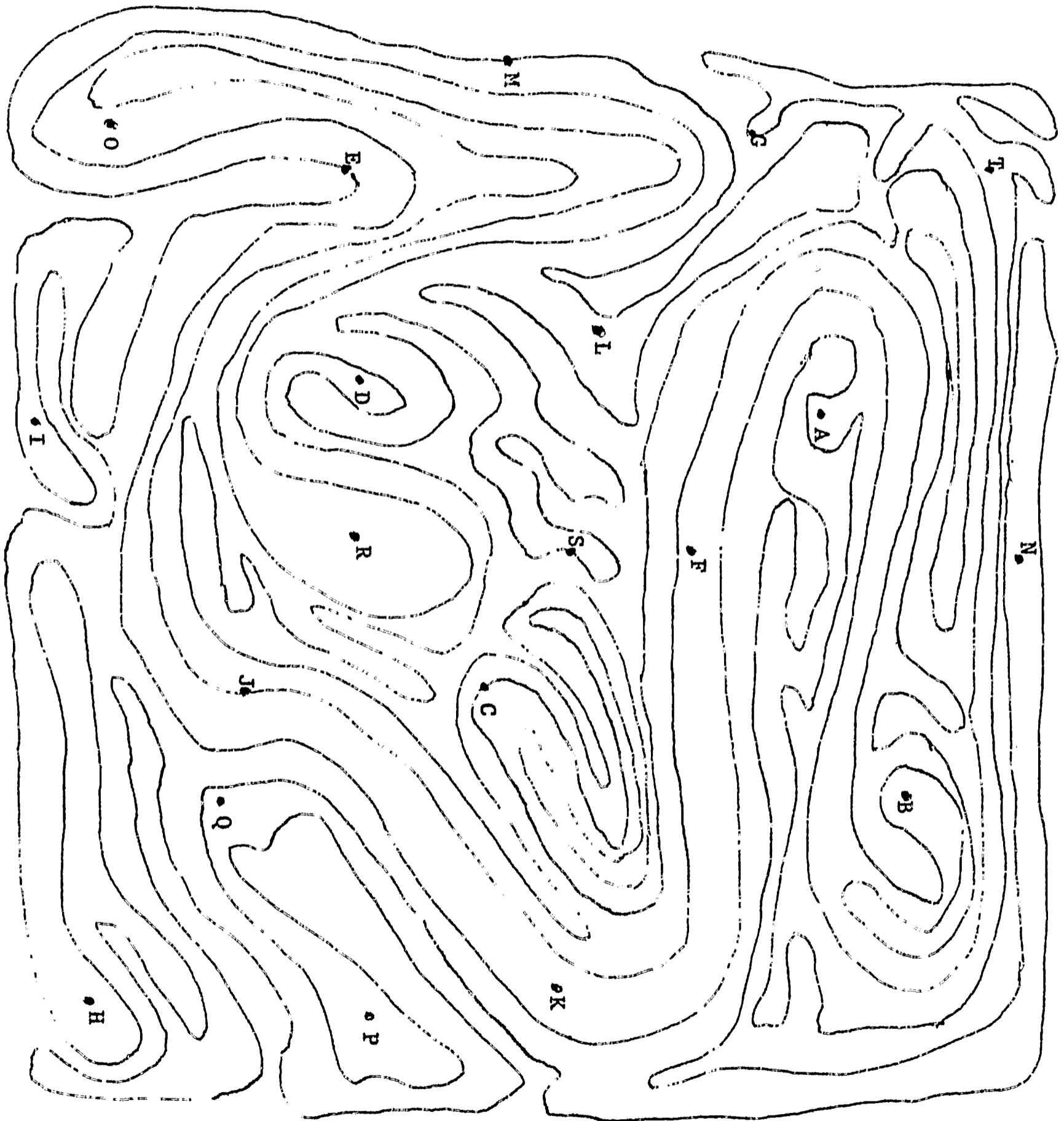
REFERENCE

Bergamini, David and Editors of LIFE. LIFE SCIENCE LIBRARY MATHEMATICS; The Silver Burdett Co., 1963.

IN, INSIDE OR OUTSIDE?

Which points are considered to be inside the figure? Outside the figure?

In the figure?



<u>IN</u>	<u>INSIDE</u>	<u>OUTSIDE</u>
C	A	B
E	H	D
G	I	F
J	L	K
S	M	O
	N	P
	T	Q
		R

General rule to follow for determining -- from a point draw a straight line, in any direction, to the outside of the figure, then counting the number of lines this straight line intersects determines whether the point is inside or outside. If the number of lines intersected is odd then the point is inside the figure, if the number of intersections is even, then the point is outside the figure. To determine those points in the figure is obvious as long as it is stated that a point in a figure is a subset of the figure itself.

Note: This idea can be done with any closed simple polygon.

Robert C. Madison

867

SIMPLE CRYPTOGRAMS

PURPOSE

To stimulate additional interest, obtain some variety, participation by students who are not too well motivated to do ordinary work, or a change of pace, but also an opportunity to progress in the idea of substitution of equals for equals, or pointing out the difference in symbols and what the symbols represent, such as numeral and number.

METHOD

Use any symbols at all to represent letters on a one-to-one correspondence, and put them together to form words in code sentences. For example, the sentence, "If you succeed in deciphering this message you receive an A for the day" could be written using numbers or other symbols as follows:

סחטו כנחאדע<חנהר נה צנדנ/ם /> \ ש
\ ג ד ו א < Δ [L V ד-חלד Δ /> \ רכלססד

Start with short easy code sentences and work them out together with the class. Give clues such as one or two letters or words, the fact that vowels are used more than consonants, 'e's are usually used more than other vowels, A or I would be one-letter words, etc.

VARIATIONS

- a. Hand out a cryptogram and say nothing -- let the students see what they can come up with by themselves.
- b. Use different symbols for letters.
- c. Ask the class to form their own code symbols and make sentences to try to stump the class or the teacher. Some will do this (if only to get even with the teacher), and several cryptograms they make will be very hard ones. For example, some of my students inserted an extra meaningless symbol between the meaningful symbols, or spelled each word backwards, or wrote the entire sentence or paragraph backwards.
- d. Give them a subtraction problem using letters for digits and ask them to try to solve it, such as PRESTO - HOCUS = FOCUS. There is only one answer to this problem.

REFERENCES

- Zimmerman, Joseph T. LAMP (Low-Achiever Motivational Program).
Des Moines Public Schools, 1966
- Dolciani, Mary P. et al. MODERN ALGEBRA, STRUCTURE AND METHOD. Houghton
Mifflin Company, 1962
- Kraitchik, Maurice. MATHEMATICAL RECREATIONS. Dover Publications, Inc. 1953
- Ball, W. W. Rouse. MATHEMATICAL RECREATIONS AND ESSAYS. The Macmillan Co.
1956

CRYPTOGRAPHY

PURPOSE

Just for fun and to acquaint ninth grade pupils with the term "Cryptography."

METHOD

A modern use of letter symbols for numbers has become established in cryptology, the science of constructing and deciphering coded messages. Though the intelligence service uses numbers for letters, some private businesses use letters for numbers on price tags to code the initial cost of goods.

Suppose a store used the letters of the word davenports for the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, respectively. An X indicates that the digit preceding it should be repeated. Find the initial cost of the following articles:
Coat: \$89.95 (NTNS); Hat: \$12.95 (RSX); Refrigerator: \$625 (VANXX).

Examine the code table and the decoded messages at the right.

0 1 2 3 4 5 6 7 8 9
a b c d e f g h i j
k l m n o p q r s t
u v w x y z

8 5 7 8 3 6 == 7 4 7 4
S P R I N G I S H E R E

1 8 2 9 4 7 4 == 4 0 7 8
V I C T O R Y I S O U R S

Any letter or digit in one of the columns may stand for any other letter or digit in the same column.

To decode a message, list all possible replacements under each numeral, then select those letters that form words. Can you decipher these messages?

1. 7083=249
2. 54024=507024039
3. WYNOI=PAD

REFERENCE

Dolciani, Berman, Freilich. MODERN ALGEBRA, BOOK 1. Houghton Mifflin Co.

Hazel V. Threlkeld

PROJECTS

PURPOSE

Motivation and to make a more pleasing but learning atmosphere.

METHOD

Bulletin Board

Use detergent boxes or plastic containers.

This year let's ALL join the trend to turn the tide toward being a better student.

Be cheerful and you can receive much joy from doing marvelous and fabulous work -- everyone duz !

PURPOSE

Motivation - each child is in competition with himself and not necessarily with the rest of the class. Help give child feelings of achievement and reward.

METHOD

Math Club

Each child becomes a member of the Math Club if he receives the same grade or a higher grade on a test compared to the previous test. This puts him in competition with himself and not necessarily with the rest of the class. A tag with each child's name could be displayed somewhere in the room. A colored mark on it could indicate how many times he has been in the club.

PURPOSE

To enhance skills and competency and make for a better learning atmosphere.

METHOD

Monthly Games or Friday Games (depending upon on how often you want to do it)

Have each child fold his paper and mark it off as the one on the following page. This is for speed and accuracy. Put three problems on the board, which each child will copy and work. The first one finished receives the number of points of how many students there are in the group (24 students, first one done gets 24 points) for each correct answer.

Second student does gets 20 points for each correct answer. When a child finished brings his paper to a designated spot, you place the number on it that he will receive for each correct answer, (10 as shown below). After all papers are turned in, you correct (by showing correct answers) and return to student who then multiplies number of correct answers times how many points he gets for each one. He then adds 10 points as a bonus if he got all 3 problems correct. Proceed and do the second round as you did the first. This time if a student got all 3 correct, he gets a bonus of 20 points. Now he totals his 2 partial scores. If he got all 6 problems correct, he adds another bonus of 100 points.

<p>10</p> <p>I. (1) $\begin{array}{r} 462 \\ 314 \\ + 28 \\ \hline 804 \\ \text{(correct)} \end{array}$</p>	<p>(2) $\begin{array}{r} 91 \\ 633 \\ 4 \\ + 190 \\ \hline 15 \\ 1,035 \end{array}$</p>	<p>(3) $\begin{array}{r} 4,263 \\ 1,724 \\ +18,004 \\ \hline 723 \\ 24,744 \end{array}$</p>	<p>I. $\begin{array}{r} 10 \\ \times 1 \\ \hline 10 \end{array}$</p>
<p>5</p> <p>II. (1) $946+231=$ $\begin{array}{r} 1,177 \\ \text{(correct)} \end{array}$</p>	<p>(2) $\begin{array}{r} 8,743 \\ 9,261 \\ +7,342 \\ \hline 25,346 \\ \text{(correct)} \end{array}$</p>	<p>(3) $\begin{array}{r} 89,621 \\ 32,480 \\ 10,947 \\ +25,003 \\ \hline 158,051 \\ \text{(correct)} \end{array}$</p>	<p>II. $\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \\ +20 \\ \hline 35 \end{array}$</p>
	<p>III. Grand Total</p> <p>$\begin{array}{r} 10 \\ 35 \\ \hline 45 \end{array}$</p>		

PURPOSE

Motivation, enhance skills and competency.

METHOD

Musical Chair Problems

Place a problem on each desk. Each child takes his pencil and paper with him and upon a certain call and in a designated order, he moves about the room and works all the problems.

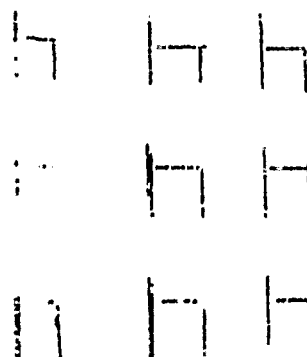
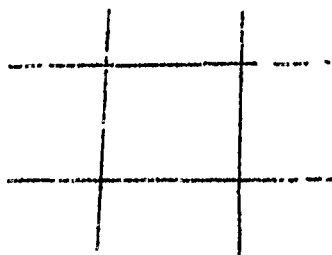
PURPOSE

Motivation and enhance skills and competency.

METHOD

Tic Tac Toe

This could be done on the board with the old familiar grid or with 9 chairs. Have 2 teams and alternate from one to the other. If they answer the question correctly they put their team's mark on the grid. If they are sitting on chairs, it pretty much has to be girls versus boys.



The game is won when one team has 3 people or team marks in a row. This also involves strategy in deciding where to place the mark or where to sit.

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PURPOSE

Motivation and enhance skills and competency.

METHOD

Baseball Game

Divide class into 2 teams.

Designate places in room for bases.

One question at a time is given to one team until they make one mistake then questions are given to the other team. When the first question is answered, the person goes to 1st base. When the next person answers the next question, he goes to 1st base and the other one on 1st base proceeds to 2nd base. Hence, a point is obtained only after 4 correct answers.

OR

Designate questions in difficulty as 1st, 2nd, 3rd or Home Run questions. The player proceeds accordingly then to the level of question he chose.

PURPOSE

In a fun way to enhance skills and competency.

METHOD

Flash Card Drill Game

One child starts by standing by another desk at the front of the room. Both see which one can give the correct answer first. The winner then gets the card and proceeds to the next desk for another question. This could be for answers to questions, problems to be solved in their head without paper definitions, etc.

PURPOSE

Motivation and enhance skills and competency.

METHOD

Math Terms A to Z

See if the students can recall math terms that begin with each of the letters of the alphabet. This may be done with the class as a whole or in a game type situation.

Sarah Tilotta
867

PROBABILITY

PURPOSE

To increase or improve student's ability to compute percentage and construct graphs.

METHOD

Consider tossing a coin 20 times. If it comes down heads, we record a 1 in the box corresponding to the number of the toss. If it comes down tails, we record a 0. Here are the results of one experiment.

No. of Tosses	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Result of Tosses	0	0	1	1	1	0	1	1	0	1	0	1	1	1	1	0	1	1	0	1
Cumulative No. of Heads	0	0	1	2	3	3	4	5	5	6	6	7	8	9	10	10	11	12	12	13
Cumulative % of Heads	0	0	33	50	60	50	57	62	55	60	54	58	61	64	66	63	64	66	63	65

In the experiment, we have 13 heads out of 20 tosses. Thus, the per cent of heads is $13/20 = .65 = 65\%$.

EXERCISES

- Toss a coin twenty times and record the number of heads and calculate the fraction of heads to the total number of tosses.
- Toss a coin 100 times. Record 1 for heads and 0 for tails. Calculate the fraction of heads to the number of tosses. Make a chart as above with percentages recorded.

In exercise A, the per cent of heads will be between 25% and 75%.

In exercise B, the per cent of heads will be between 35% and 65%.

ADDITIONAL EXERCISES

Have students plot graphs showing cumulative per cent of times coin landed "heads" or thumbtack landed "up." One way might be a line graph with "tosses" along bottom of graph and per cent up side.

VARIATIONS

Thumbtacks may be used, and number landing up can be recorded "cumulative number up" in table.

Norman G. Wolfe

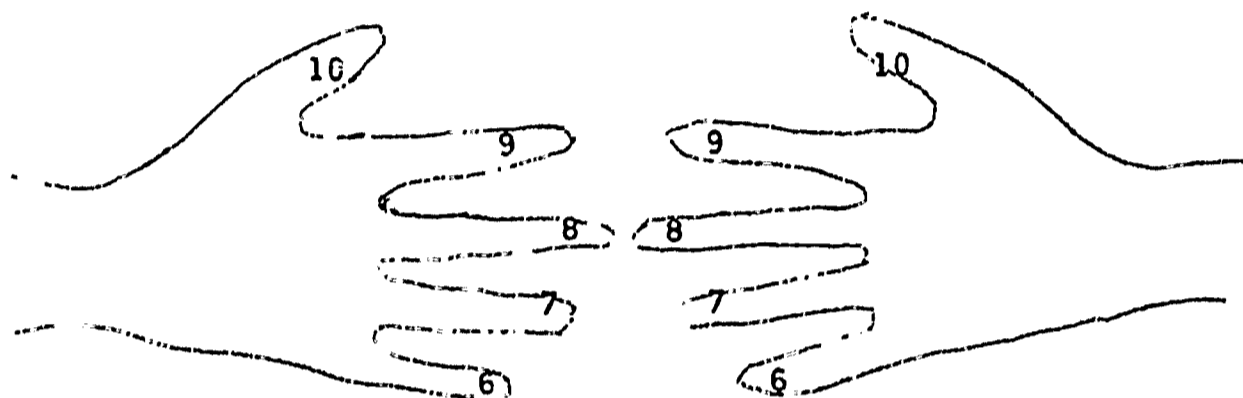
FINGER MULTIPLICATION

PURPOSE

To help those children who are having difficulty with multiplication facts for sixes through nines.

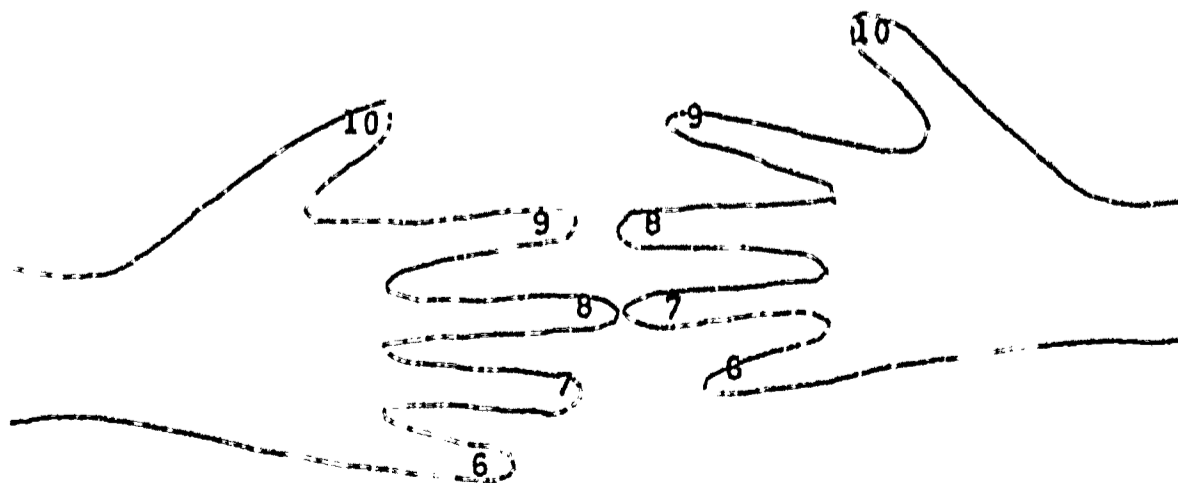
METHOD

The fingers of each hand are numbered from 6 through 10, with the little finger on each hand representing the number 6, and the thumb on each hand representing the number 10.



To multiply 8×8 , let the two fingers which represent 8, touch. Find the number of fingers below the touching fingers (four), and add the two fingers that touch for a total of six. This tells the number of tens in the product of 6×10 . Multiply the number of fingers on each hand above the touching fingers to set the number of ones in the product. There are two fingers on the right hand and two fingers on the left hand above the touching fingers, $2 \times 2 = 4$. So, $8 \times 8 = 60 + 4$, or 64.

For 8×7 , there are three fingers that are below the touching fingers plus two fingers that touch for a total of five. So, the number of tens in the product is 5×10 , or 50. The number of fingers on the left hand above the touching finger is two, while the number of fingers on the right hand above the touching finger is three. Then the number of ones in the product is 2×3 , or 6. So $8 \times 7 = 50 + 6$, or 56.



REFERENCES

Arithmeric Games and Activities, by Wagner, Hosier and Gilloley,
Teachers Publishing Corporation, Darien, Connecticut, 1964.

Fern P. Johnson

867

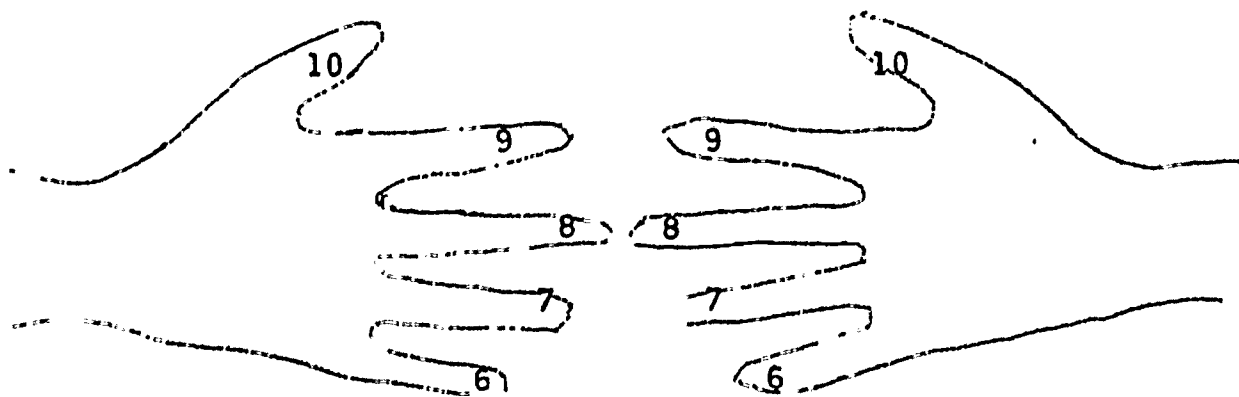
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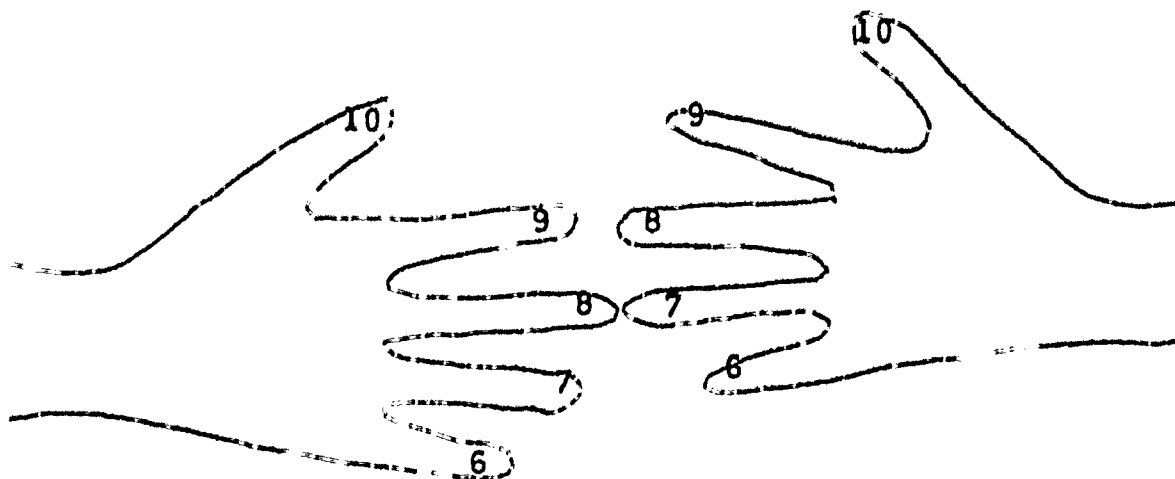
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REFERENCES

Arithmeric Games and Activities, by Wagner, Hosier and Gilloley,
Teachers Publishing Corporation, Darien, Connecticut, 1964.

Fern P. Johnson

867

DAYS IN A MONTH

PURPOSE

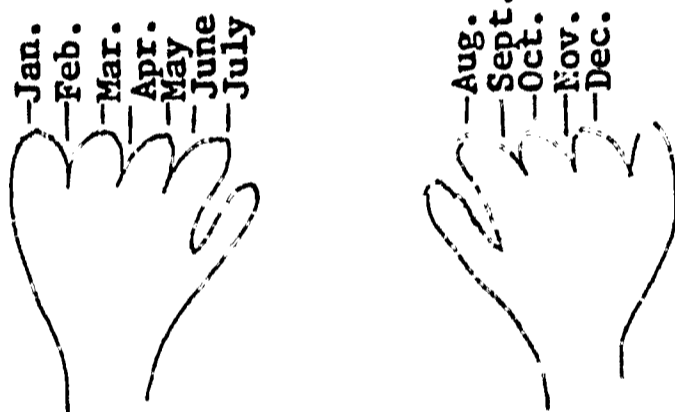
To give the student an easy way of remembering the number of days in a month.

METHOD

The following limerick is useful in remembering the number of days in each month.

Thirty days has September,
April, June and November.
All the rest I can't remember.
Why bother me at all
When the calendar hangs on the wall?

Another useful tool for remembering the days in each month can be found on the knuckles of your hand. (See below)



Note that each knuckle corresponds to a month with 31 days while each space between the knuckles corresponds to a month with 30 days. (With the exception of February)

James I. Reed

867

PRODUCTS

(Factors whose sum ends in 0)

PURPOSE

To develop a short cut for multiplying numbers whose average is a multiple of 10. (i.e., 12×8 , 25×35 , etc.). Let the student discover the pattern if possible.

METHOD

Begin with the question, "What is 12×8 ?" Most of the students will multiply it out the traditional way. Ask them for relationships between the two numbers. (Look for answers indicating that their sum is a multiple of 10). Then ask them to rewrite 12 and 8 using the numeral 10. When they have correctly done this, $[(10+2) \text{ and } (10-2)]$ ask them to multiply using the distributive property. Have a student do this problem on the board. It becomes $(10+2)(10-2) = (10 \times 10) + (2 \times 10) - (2 \times 10) = 2 \times 2$
 $= 100 + 20 - 20 - 4$
 $= 100 - 4$
 $= 96$

Now ask the students the product of 25 and 35. Suggest they use the above procedure when obtaining their answer. Have a student put his solution on the board.

$$25 \times 35 = (30-5)(30+5) = 900 - 25 = 875$$

Finally, ask for the generalization. You may have to do more examples than this before asking for the general rule.

James I. Reed

SQUARING FIVES

PURPOSE

To develop a rule for quickly determining the square of any number ending in 5. This should be developed in class via a discovery approach by the students to the extent that this is possible.

METHOD

Begin by putting the number 15 on the board. Ask the students how they might rewrite 15 so as to make it easier to square it. (Hint that you wish to use the distributive property. When they suggest $15 = 10 + 5$, write the following:

$$\begin{aligned} 15^2 &= (10 + 5)(10 + 5) \\ &= (10 \times 10) + 2(10 \times 5) + (5 \times 5) \\ &= 100 + 2(50) + 25 \\ &= 100 + 100 + 25 \\ &= 200 + 25 \end{aligned}$$

Now ask them how they might rewrite 20 so as to write it as the product of two multiples of 10. They will soon tell you it can be written as 10×20 . Then, we have $20^2 = (10 \times 20) + 25 = 225$.

Continue the above procedure with 25. They should note that $25^2 = (20 \times 30) + 25 = 625$. Ask them for a general rule. If they fail to see it, continue the above procedure with 35. Note that $35^2 = (30 \times 40) + 25 = 1225$.

Continue to perform the above steps until the students reach the decision that any square of a number ending in 5 is the product of 5 more and 5 less than the number plus 25.

Be sure to have them check the rule for larger numbers such as 95^2 and 115^2 .

James I. Reed

GAMES FOR DEVELOPING SKILLS

PURPOSE

Oral work in simple addition, subtraction, and multiplication of whole numbers or in expressing fractions in simplest form or in adding fractions where denominators are the same.

METHOD

Variation of Spelling Bee

Divide the class into two teams. Ask a question of one member. If he answers the question correctly, he earns two points for his team. If he cannot answer the question, the child on either side of the original person questioned gets an opportunity to answer the question. If either child is successful, he earns one point for his team. If not successful, the question goes to the opposing team.

Another variation of a spelling bee begins with all pupils standing. During the first and second round anyone who misses must sit down. However, on the third round, an opportunity is given for those pupils sitting down to answer the questions missed by those standing. Thus they have another chance to stand. Every other round they have another chance. Thus they do not lose interest in the proceedings.

PURPOSE

Practice in the four operations in whole numbers, in fractions, in recalling common fraction and per cent equivalents, and in reviewing the relationship between measuring units.

METHOD

Number Lotto

Lotto cards are made of square pieces of cardboard. On each card there are placed 3 rows of 3 numbers each. Each player is given a card and he provides himself with covers. A caller holds up a flash card with a number combination and if a player has the correct answer on his card, he covers the number. The first player to cover all nine of his numbers is the winner, and becomes the next caller.

PURPOSE

Practice in addition and multiplication with whole numbers and fractions.

METHOD

Spinning Game

On large white cardboard, 20 x 26, is drawn a circle with a 12" radius. Inside the large circle is drawn a smaller circle with a radius of 4". The larger circle is divided into nine sections, each representing a different number. In the center of the smaller circle is placed a wire with a pointer at one end. The class is divided into several teams with each player on the team being allowed two spins each. The player finds the product or the sum of the two numbers he spins to get his individual score. Each team totals the score of its individual players to reach the team score. The team with the highest score wins.

PURPOSE

Forming and reading whole numbers, decimal fractions, and mixed numbers.

METHOD

Merry Mix-Up

Each of two teams is supplied with a set of eleven flash cards. These cards each contain one of the ten digits and a decimal point. The teacher names a number such as 128, 5.06, or .097. The teacher and the other pupils who are not involved in the game decide which team has formed the numeral first. The teams can also be asked to name the largest or the smallest number possible with designated digits.

PURPOSE

Practice in vocabulary associated with mathematics.

PURPOSE

Stump the Experts

A panel of six pupils sits in front of the class. The teacher shows the class but not the panel a word used in Mathematics. The panel tries to discover the word by asking the class questions which can only be answered by "yes" or "no." For example, "Is this word associated with Multiplication?" "Is this the name of a plane figure?" If the panel has not discovered the word in ten questions, a new panel is formed. A panel remains in the front of the class until it is "stumped."

METHOD

Practice in operations with whole numbers, common fractions, decimal fractions per cent equivalents, finding the factors of a number, or selecting primes.

PURPOSE

Around the World

The first two youngsters in the first row have a contest to see who can answer the flash card combination first. The winner goes on to the next person to compete with him. If a child is the loser, he remains at the place or seat where he lost. Only by winning can he move on. He tries to get around the class and back to his seat again. Time does not always permit this. Then the winner is the child who has traveled farthest from his seat.

June Meeks

867

ADDITION

PURPOSE

To provide opportunity to review column addition and possible short cuts they may or may not recognize.

To provide an assignment in which all can probably do well.

To improve accuracy in addition.

To discover a way in which the student can prove to themselves and others that their answers are correct.

METHOD

Approach to the assignment.

Warmup session including rapid but easy board drill. It is helpful to begin the class period by having 6 or 8 pupils demonstrate at the board. Dictate several short columns of numbers. Encourage them to add rapidly. Encourage them to combine as many numbers into repeated combinations as they can recognize. Seventh grade pupils will have to be helped to discover the various combinations in the first two or three problems. (See following samples. It is best if the teacher builds a few of his own so that he can include the combinations which he has discovered they do not seem to remember.)

1. Hand out the individual copies prepared before class.
2. Allow pupils a few moments to look at the sheet.
3. Explain that each row of numbers is treated as a separate addition problem and to place the answer on the line provided at the bottom of each row.
4. Consider each row of numbers as they are arranged horizontally on the paper. Pretend that you placed a plus sign between each number in the row. Add the numbers and place the answers for each row on the lines provided for them.
5. Add each vertical row of numbers. Place answers on lines at the bottom of each row.
6. Add the answers you found for each separate row when adding each horizontal column. Save this answer to use in comparing later.
7. Add the answers you found for each separate row when adding each vertical column.
8. Compare the answer you found for number five with the answer you found for number six. If they are the same you have not made a mistake on any of the columns.
9. Clue-- Your answer should be between 500 and 1000.

VARIATIONS

Depending on the time for class preparation that the teacher might have, it probably would be fine if one or two others could be developed which would contain frequently missed errors which the pupils seem to make.

Probably the better groups could develop some of their own which they in turn could challenge their fellow class members.

REFERENCES

I have this exercise in my files under the heading of 'materials good for first week.

APPROACH TO ASSIGNMENT

Samples to use for board drill:

$$\begin{array}{r} 7 \overline{)9} \\ 2 \overline{)9} \\ 8 \overline{)9} \\ 1 \overline{)9} \\ 9 \overline{)9} \\ 0 \overline{)9} \\ 6 \overline{)9} \\ 3 \overline{)9} \\ 7 \overline{)9} \\ 2 \overline{)9} \\ 5 \overline{)9} \\ 4 \overline{)9} \\ \hline 54 \end{array}$$

Solution
 $6 \times 9 = 54$

$$\begin{array}{r} 6 \overline{)11} \\ 5 \overline{)11} \\ 7 \overline{)11} \\ 4 \overline{)11} \\ 6 \overline{)11} \\ 5 \overline{)11} \\ 8 \overline{)11} \\ 3 \overline{)11} \\ 9 \overline{)11} \\ 2 \overline{)11} \\ 5 \overline{)11} \\ 6 \overline{)11} \\ \hline 66 \end{array}$$

Solution
 $6 \times 11 = 66$

$$\begin{array}{r} 7 \overline{)10} \\ 2 \overline{)10} \\ 1 \overline{)10} \\ 6 \overline{)10} \\ 2 \overline{)10} \\ 2 \overline{)10} \\ 3 \overline{)10} \\ 5 \overline{)10} \\ 2 \overline{)10} \\ 4 \overline{)10} \\ 4 \overline{)10} \\ 2 \overline{)10} \\ \hline 54 \end{array}$$

Varied pupils
will see
 $11 + 9 = 20 \times 3 = 60$
or
 $3 \times 9 = 27$
 $3 \times 11 = 33$
 $\underline{60}$

$$\begin{array}{r} 5 \overline{)9} \\ 4 \overline{)9} \\ 6 \overline{)11} \\ 5 \overline{)11} \\ 9 \overline{)9} \\ 0 \overline{)9} \\ 7 \overline{)11} \\ 4 \overline{)11} \\ 8 \overline{)9} \\ 1 \overline{)9} \\ 5 \overline{)11} \\ 6 \overline{)11} \\ \hline 60 \end{array}$$

$$\begin{array}{r} 2 \overline{)9} \\ 3 \overline{)9} \\ 4 \overline{)9} \\ 4 \overline{)9} \\ 3 \overline{)9} \\ 2 \overline{)9} \\ 2 \overline{)9} \\ 3 \overline{)9} \\ 4 \overline{)9} \\ 4 \overline{)9} \\ 3 \overline{)9} \\ 2 \overline{)9} \\ \hline 36 \end{array}$$

$$\begin{array}{r} 5 \overline{)11} \\ 6 \overline{)11} \\ 6 \overline{)11} \\ 5 \overline{)11} \\ 4 \overline{)11} \\ 7 \overline{)11} \\ 7 \overline{)11} \\ 4 \overline{)11} \\ 3 \overline{)11} \\ 8 \overline{)11} \\ 8 \overline{)11} \\ 3 \overline{)11} \\ \hline 0 \end{array}$$

1) Chal-
1) lenge
0
 $\underline{68}$ What to
do here?

$$\begin{array}{r} 6 \times 11 = 66 \\ + 2 \\ \hline 68 \end{array}$$

Material to be copied in such a manner as to provide each pupil with an individual copy.

Addition

2	4	6	8	3	5	1	7	9	_____
1	3	5	7	9	4	6	2	8	_____
8	9	7	9	7	5	6	8	4	_____
4	6	2	8	3	5	9	5	7	_____
7	5	4	8	3	9	4	7	8	_____
9	7	4	3	5	4	6	7	4	_____
9	7	7	6	4	5	7	9	4	_____
2	5	7	9	6	4	3	7	9	_____
8	6	5	7	4	3	8	6	5	_____
9	8	6	4	5	3	6	8	4	_____

_____ Proof

Directions

Add across each horizontal column, and down each vertical column. Add the answers that you found for the horizontal columns. Add the answers that you found for the vertical columns. Compare the two grand total answers. Your proof will be that you found the same answer for each grand total.

KEY FOR THE TEACHER

Answers for vertical rows: 59 60 53 69 49 47 56 66 62

Answers for horizontal rows: 45 45 63 49 55 49 58 52 52 53

Proof answer: 521

Elsie E. Sawyers

867

THE SNAIL AND THE WELL

PURPOSE

To point out the necessity of investigating carefully all data given. Also to understand the question precisely.

METHOD

A snail finds himself at the bottom of a thirty foot well. If he climbs up three feet and slides back two feet in each 24 hour day, how many days will it take him to reach the top of the well?

ANSWER: 28 days.

EXPLANATION OF ANSWER:

After 27 days the snail has reached 27 feet. On the 28th day he climbs 3 feet and has reached the top of the well before he slides back. Since the question asks when he will reach the TOP of the well, the 28th day must be the correct answer.

REFERENCES

Adler, Irving. MAGIC HOUSE OF NUMBERS. The New American Library of World Literature, Inc., New York, New York.

Hunter, J. A. H. FUN WITH FIGURES. Dover Publications, Inc., New York, New York.

Contains bibliography of many fun books.

Hunter, J. A. H. MATH BRAIN TEASERS. Bantam Books, Inc., New York, New York.

Mott-Smith, Geoffrey. MATHEMATICAL PUZZLES. Dover Publications, Inc., New York, New York.

William H. Shay

PRODUCTS

(Factors whose sum ends in 0)

PURPOSE

To develop a short cut for multiplying numbers whose average is a multiple of 10. (i.e., 12×8 , 25×35 , etc.). Let the student discover the pattern if possible.

METHOD

Begin with the question, "What is 12×8 ?" Most of the students will multiply it out the traditional way. Ask them for relationships between the two numbers. (Look for answers indicating that their sum is a multiple of 10). Then ask them to rewrite 12 and 8 using the numeral 10. When they have correctly done this, $[(10+2)$ and $(10-2)]$ ask them to multiply using the distributive property. Have a student do this problem on the board. It becomes $(10+2)(10-2) = (10 \times 10) + (2 \times 10) - (2 \times 10) = 2 \times 2$
 $= 100 + 20 - 20 - 4$
 $= 100 - 4$
 $= 96$

Now ask the students the product of 25 and 35. Suggest they use the above procedure when obtaining their answer. Have a student put his solution on the board.

$$25 \times 35 = (30-5)(30+5) = 900 - 25 = 875$$

Finally, ask for the generalization. You may have to do more examples than this before asking for the general rule.

James I. Reed

867

SQUARING FIVES

PURPOSE

To develop a rule for quickly determining the square of any number ending in 5. This should be developed in class via a discovery approach by the students to the extent that this is possible.

METHOD

Begin by putting the number 15 on the board. Ask the students how they might rewrite 15 so as to make it easier to square it. (Hint that you wish to use the distributive property. When they suggest $15 = 10 + 5$, write the following:

$$\begin{aligned} 15^2 &= (10 + 5)(10 + 5) \\ &= (10 \times 10) + 2(10 \times 5) + (5 \times 5) \\ &= 100 + 2(50) + 25 \\ &= 100 + 100 + 25 \\ &= 200 + 25 \end{aligned}$$

Now ask them how they might rewrite 20 so as to write it as the product of two multiples of 10. They will soon tell you it can be written as 10×20 . Then, we have $15^2 = (10 \times 20) + 25 = 225$.

Continue the above procedure with 25. They should note that $25^2 = (20 \times 30) + 25 = 625$. Ask them for a general rule. If they fail to see it, continue the above procedure with 35. Note that $35^2 = (30 \times 40) + 25 = 1225$.

Continue to perform the above steps until the students reach the decision that any square of a number ending in 5 is the product of 5 more and 5 less than the number plus 25.

Be sure to have them check the rule for larger numbers such as 95^2 and 115^2 .

James I. Reed

867

THE GOAT IN THE FIELD

PURPOSE

To apply the knowledge of finding the area of circles to any applied problem.

METHOD

1. Provide a copy of the problem for each student or write the problem on the chalkboard.
2. Suggest that a diagram be drawn to help visualize the situation.
3. Allow ample time for the class to work the problem.
4. Lead the class to see the need for stating a value for π and the need for determining the degree of precision for the answer.

VARIATIONS

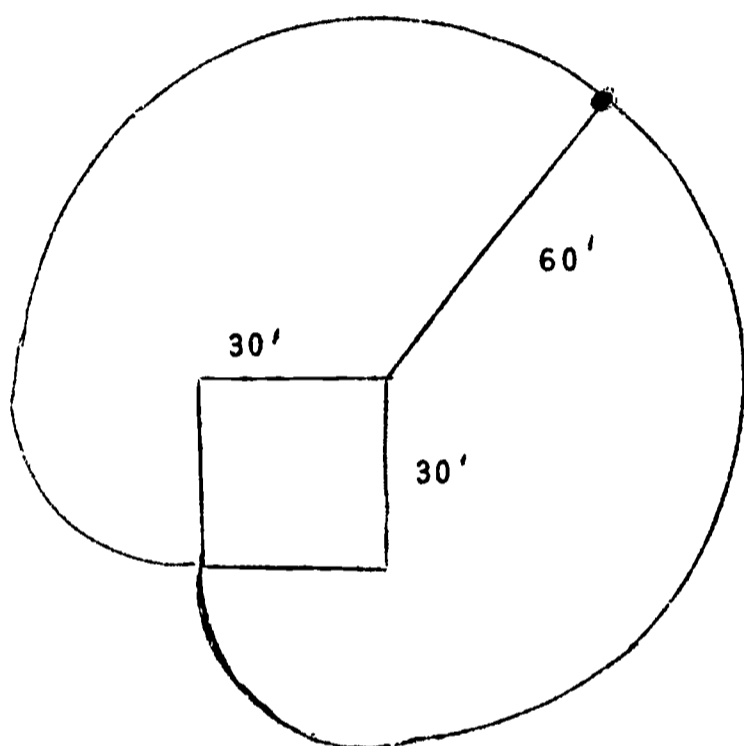
This could also be used after studying precision in measures and finding the areas of circles.

REFERENCE

Longley-Cook, L. H. WORK THIS ONE OUT; Fawcett Publications, Inc., 1963, pp. 41-42; Problem No. 69.

THE GOAT IN THE FIELD

A square shack 30 feet by 30 feet is in the middle of an open field. A goat is tethered to one corner of the shack, by a chain 60 feet long. The goat cannot get under the shack but can graze anywhere else as the chain will allow. What is the area of the portion of the field that the goat can graze?



$$A = \pi r^2$$

A_1 = Area of large portion

$$A_1 = 3/4 (\pi 60^2)$$

$$A_1 = 3/4 (3600) \pi$$

$$A_1 = 2700 \pi$$

A_2 = Area of two small portions

$$A_2 = 1/2 (\pi 30^2)$$

$$A_2 = 1/2 (900) \pi$$

$$A_2 = 450 \pi$$

TA = TOTAL AREA

$$A_1 + A_2 = TA$$

$$2700 \pi + 450 \pi = TA$$

$$3150 \pi = TA$$

Robert C. Madison

867

UNION vs INTERSECTION

PURPOSE

Low achievers quite often have trouble understanding the union and intersection of line segments even though they may know the meaning of a union set and intersection set. This classroom procedure involves the students and also helps form a better picture in their minds of the segment representing the union set or intersection set.

METHOD

Have four students volunteer to come up before the class. Each student represents a point on a line segment. Two people will represent end points and the others points in between the end points of the segment. Keep the students in a straight line by having them stand on one of the lines in the floor of the room.

Example: Danny, Kathleen, Terry and Linda volunteer to be a part of the demonstration. Draw a line segment and mark four points on it representing the demonstration. Use the first letter of each name to represent the four known points on the segment.



$$\overline{DT} \cap \overline{KL} = \overline{KT}$$

$$\overline{DK} \cap \overline{KT} = \{K\}$$

$$\overline{DT} \cup \overline{KL} = \overline{DL}$$

$$\overline{DK} \cup \overline{KT} = \overline{DT}$$

VARIATION

This demonstration may be used on lines, rays, line segments or any combination of the two.

REFERENCE

The above gimmick is my own creation.

Jay Mills

867

FOUR SQUARE PROBLEM

PURPOSE

To develop a thought process. To strengthen the definition of a square and to divert from the routine of the regular classroom situation.

METHOD

1. Either provide a copy of the diagram for each student or hand out sixteen (16) matches or toothpicks to form the given diagram.
2. Ask students to move only two (2) sticks to form four squares, each touching another.

VARIATIONS

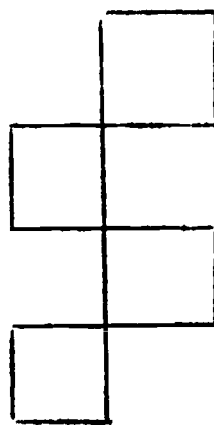
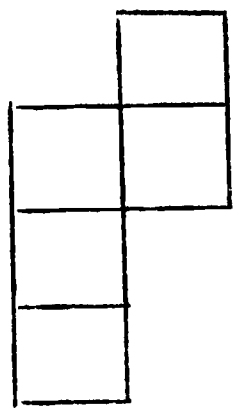
This could be used when there is an extra five or ten minutes before class dismissal.

REFERENCE

MATHEMATICS STUDENT JOURNAL, Problem No. 150 (November 1960)

FOUR SQUARE PROBLEM

Set up with matches, the five-square formation as shown below. By moving just two matches, form four squares, each touching another, instead of five squares.



Answer

Robert C. Madison

A TAKE HOME PROBLEM

PURPOSE

Primarily for interest. Also experience in area and volume.

METHOD

The problem given to the student at the end of the class period so that the student will more likely think for himself instead of quickly guessing the correct answer and getting verification from the teacher. Answer passed out first thing the next class period.

VARIATIONS

Presenting it with more explanation.

Here is the problem:

Let's have fun.

These are the facts:

- a. A DAKE is exactly four SMAGS long.
- b. A QUATTOL holds water; so does a TOLQUAT.
- c. The top of a QUATTOL measures 1 SMAG by 1 SMAG.
- d. The top of a TOLQUAT measures 1 DAKE by 1 DAKE.
- e. A QUATTOL is one DAKE in depth.
- f. A TOLQUAT is one SMAG in depth.

These are the questions:

1. If both the QUATTOL and the TOLQUAT are box-shaped with parallel sides, which one holds more water?
2. How much more water does the one hold than the other?

Answer:

1. The TOLQUAT holds more water. Here is the reasoning: A QUATTOL is 1 DAKE deep and a DAKE is equivalent to 4 SMAGS; so a QUATTOL holds 4 cubic SMAGS of water. The top of the TOLQUAT (1 DAKE by 1 DAKE) is 4 SMAGS by 4 SMAGS, or 16 square SMAGS; the TOLQUAT is just 1 SMAG in depth; therefore it holds 16 cubic SMAGS of water.
2. The TOLQUAT holds 4 times as much water as the QUATTOL.

Martha Wilkins

867

MATH PUZZLERS

PURPOSE

To use mathematical skills and the same time have fun.

METHOD

Do it together as a class. Discuss why it works.

VARIATIONS

Try it on one another.

REFERENCES

Tocquet, Robert: The Magic of Numbers

- I. Multiply the first number of your age by 5. Add 3. Double this figure. Add the second number of your age. Have him give you this answer. Now subtract 6 and tell him his age.
- II. Write the number of the month in which you were born. January is no. 1. Add the next higher number. Multiply by 50. Add the number of the months in a year. Add the day of the month of your birthday. Multiply by 100. Add the number of weeks in a year. Add the number formed by the last two figures in your birth year. Now ask for this number and then subtract 6252. This tells the birth date, for example 50746 means May 7, 1946.
- III. A snail crawling up a pole 10 feet high climbs 3 feet every day and slips back 2 feet every night. How long will it take to reach the top?
- IV. A woman had 5 pieces of gold chain, each having 3 links. She ask a jeweler how much he would charge to make them into one chain. The jeweler replied that he charged 25¢ to cut a link and 25¢ to weld a link. How much would the charge of the jeweler be?
- V. Seven paperbound volumes of a book are on a shelf in regular order. A bookworm bored a hole straight through the front cover of Volume I to the back cover of Volume VII. Each book was 1 inch thick. How long a tunnel did the worm bore?
- VI. A woman went to a well with two jars, one of which held 3 pints and the other 5 pints. She brought back just 4 pints of water. How did she manage to do it?

- VII. If you received 10¢ on your first birthday, 20¢ on your second birthday, 40¢ on your third, and so on, each time receiving twice as much as the time before, how much would you have after receiving the gift on your 16th birthday (if you saved all)?
- VIII. Two fathers and two sons divided \$3.00 among themselves, each received exactly \$1.00. How was this possible?
- IX. A girl was carrying a basket of eggs. A man driving a horse hit the basket and broke all the eggs. He asked the girl how many eggs she had had. The girl replied, "When I counted them by twos there was one left over; when I counted them by threes there was one left over; when I counted them by fours there was one left over; but when I counted them by fives no eggs were left over." What was the smallest number of eggs she could have had in her basket?
- X. Two apple women each had 30 apples for sale.
- (a) If the first sold hers at the rate of 2 for 1¢, how many cents would she have received?
 - (b) If the other sold hers at 3 for 1 cent, how many cents would she have received?
 - (c) Together they would have received how many cents?
 - (d) They put the apples together and sold them at 5 for 2¢. How much did they get for the 60 apples?

THE WORM AND THE DICTIONARIES

Three dictionaries of 1,000 pages each, 3 inches thick, plus 1/8 inch each side for the binding, stand in a bookshelf. The three volumes together, therefore, form a total thickness of $9 \frac{3}{4}$ inches. A worm starts at page 1 of the first volume; it bores through the paper and cardboard and reaches page 1,000 of the third volume. What distance has it traveled?

THE ARAB'S TESTAMENT

An Arab dies, leaving his fortune of 17 camels to his three sons. He left half to the eldest, a third to the second son and a ninth to the youngest. How did the three sons divide up the camels?

Evelena Stravers
Kathleen B. Smith

ADD-A-TAIL PUZZLE

PURPOSE

A sneaky way of getting students to: (1) add; (2) to use logic.

Solution

96	82	61	11	50
31	72	61	16	74
100	20	90	60	70
90	81	10	60	59
62	70	88	20	10
				1050

96	82	61	11	50
31	72	61	16	74
100	20	90	60	70
90	81	10	60	59
62	70	88	21	10
				1050

METHOD

Copy the puzzle on another sheet of paper. The student is to draw a continuous line connecting some of the numbers in the puzzle. It must end at the number in the bottom box, and the numbers in the trail must add up to the total in the bottom box. The trail cannot cut corners. It cannot retrace itself. It cannot cross itself. You can use each box only once. There may be more than one way to make a trail.

Alta B. Cameron

867

A COMPUTING MACHINE

PURPOSE

To carry on with the idea of flow charts and to use a set of numbers which the students are already familiar with. (prime numbers)

To show the students how important it is to follow simple instructions.

To show that even though a computer can do many jobs, it must first be given a program which is the work of a human being.

METHOD

Because of the time that is needed to set up this machine, it is necessary to use two days to organize and experiment.

1st Day -- The students should be informed that they are going to play the parts of a computer, which has been programmed to generate the prime numbers. At this time each student is assigned to play one of the 23 parts of the machine. If you have more than 23 students, you might want to assign them jobs such as handing out extra paper, etc. It has been found from experience that it is a good idea to choose the better students in the class for the more complicated positions.

At this time hand out the instructions and the seating charts. Go over the general instructions in class, stressing each of the four instructions. Give the machine parts (students) ample time to study their own particular assignments and then if time permits, push the button.

Chances are that the machine will not work the first time through. I've found that failure at this point causes the students to become even more interested, rather than disappointed. Instruct the students to check over their instructions, and to be prepared to come to class the next day ready to make the machine work. It is also necessary for each student to study the seating chart, so that he will be able to follow the flow of information.

2nd Day -- Make sure that plenty of paper is available ahead of time.
Have person who is playing part of starter
Instruct Starter to push button.

VARIATIONS

If you have a good background in programming, write a program of your own.

REFERENCES

Hausner, Melvin, "On an Easy Construction of a Computing Machine," The Mathematics Teacher, April, 1966, page 351.

COMPUTERS

General Instructions:

1. Put your letter on every sheet of paper you hand out.
2. If you receive a number without a letter on it, shout "Stop"!
3. If someone shouts, "Stop," do so.
4. Do not discard any papers unless you are instructed to do so.

Parts

Start: Hand (1) to A and shout "S."

Print: Write what you receive on the blackboard.

- A: Write the number you received on two sheets of paper. Hand one to "Print." Hand one to G.
- B: Write the number you receive on five sheets of paper. Hand one to J, one to K, one to L, one to N, and one to O.
- C: Write the number you receive on two sheets of paper. Hand one to L, and hand one to O.
- D: Write the number you receive on three sheets of paper. Hand one to H, one to P, and one to U.
- E: Write the number you receive on two sheets of paper. Hand one to I and one to V.
- F: Write the number you receive on two sheets of paper. Hand one to D and one to E.
- G: Add: (1) to the number you receive. Write your answer on a sheet of paper. Hand it to B.
- H: Add (1) to the number you receive. Write your answer on a sheet of paper. Hand it to R.
- I: Add (1) to the number you receive. Write your answer on a sheet of paper. Hand it to Q.
- J: Hold on to the number labeled B until you receive another one from B. When you receive a new number, destroy the old one. If the number you receive from U is greater than B, shout " t_1 ." If not, shout " t_2 ."
- K: Hold on to the number labeled B until you receive another one from him. When you receive a new number, destroy the old one. If the number you receive from O is greater than B, shout " t_3 ." If not, shout " t_4 ."

- L: Hold on to the number labeled B until you receive another one from him. When you receive a new number, destroy the old one. If the number you receive from C is equal to B, shout " t_5 ." If not, shout " t_6 ."
- M: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_1 ," copy this number on a sheet of paper and hand it to A.
- N: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_5 ," copy this number on a sheet of paper and hand it to G.
- O: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_6 ," copy this number on a sheet of paper and hand it to K.
- P: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_2 ," copy this number on a sheet of paper and hand it to V.
- Q: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_4 ," copy this number on a sheet of paper and hand it to E.
- R: Hold on to the number you receive until you receive another one. When you receive a new number, destroy the old one. When you hear " t_3 ," copy this number on a sheet of paper and hand it to F.
- S: Hold on to the number (2). When you hear "S," or " t_1 ," or " t_5 ," copy (2) on a sheet of paper and hand it to F.
- U: Take the number you receive. Square it. Write your answer on a sheet of paper and hand it to J.
- V: Hold on to the numbers you receive from P and E until they are replaced or destroyed. If you hear " t_1 ," " t_3 ," or " t_5 ," destroy the number from P. If you have a number from P and a number from E, multiply them together, write your answer on a sheet of paper, and hand it to C. Repeat this last operation upon receipt of a new number.

WHAT'S YOUR MATHEMATIC IQ?

1. If you went to bed at 8 o'clock at night and set the alarm to get you up at 9 o'clock in the morning how many hours sleep would you get?
2. Do they have a 4th of July celebration in England?
3. How many birthdays does the average man have?
4. Why can't a man living in Winston-Salem, N. C., be buried west of the Mississippi River?
5. If you had only one match and entered a room in which there was a kerosene lamp, an oil heater, and a woodburning stove, which would you light first?
6. Some months have 30 days, some have 31, how many have 28?
7. If a doctor gave you three pills and told you to take one every half hour how long would they last you?
8. A man builds a house with four sides to it and it is a rectangular shape; each side has a southern exposure. A big bear wanders by-- what color is the bear?
9. How far can a dog run into the woods?
10. What four words appear on every denomination of U. S. coins?
11. What is the minimum number of active baseball players "ON THE FIELD" during any part of an inning? How many outs in each inning?
12. I have in my hand only two U. S. coins which total 55¢ in value. One is not a nickel. Please bear this in mind. What are the two coins?
13. A farmer had 17 sheep. All but 9 died. How many did he have left?
14. Divide 30 by $1/2$ and add ten. What is the answer?
15. Two men were playing checkers. Each played five games and each man won the same number of games. No draws. How can you figure this?
16. Take two apples from three apples and what do you have?
17. An archaeologist claimed he found some coins of gold dated 46 B.C. Do you think he really did?
18. A woman gave a beggar 50¢. The woman was the beggar's sister but the beggar is not the woman's brother. How come?
19. How many animals of each species did Moses take aboard the Ark with him?
20. Is it legal in California for a man to marry his widow's sister?
21. What word is misspelled in this test?

NAME _____

DATE _____

HERE'S A TEST JUST FOR FUN
CONCENTRATE AND REMEMBER YOU HAVE ONLY THREE MINUTES

1. Read everything before doing anything.
2. Put your name in the upper right hand corner of this paper.
3. Circle the word "name" in sentence two.
4. Draw five small squares in the upper left hand corner.
5. Put an "X" in each square.
6. Put a circle around each square.
7. Sign your name under the title.
8. After the title write "yes, yes, yes."
9. Put a circle around each word in sentence in No. 7.
10. Put an "X" in the lower left hand corner of this paper.
11. Draw a triangle around the "X" you just put down.
12. On the reverse side of this paper multiply 703 by 9805.
13. Draw a rectangle around the word "paper" in sentence No. 10.
14. Call out your first name when you get to this point in the test.
15. If you think you have followed directions up to this point, call out,
"I have."
16. On the reverse side of this paper add 9850 and 8950.
17. Put a circle around your answer to No. 16.
18. Put a square around the circle.
19. Count out loud in your normal speaking voice backwards from ten to one.
20. Now that you have finished reading the directions carefully, go back
and do only sentences one and two.

Clarence H. Lippert

NUMERICAL TIC-TAC-TOE

PURPOSE

To confront the student with a game-type situation whereby he may develop some mathematical strategies as well as become more proficient at adding and subtracting numbers.

METHOD

Begin by explaining to the students that the object of the game is to place the numbers in the lattice so that the sum in any row, column, or diagonal is 15. The class should be divided into two teams, boys against girls, one half the room against the other, or any other division which suits the teacher's fancy.

A student recorder may be chosen by the teacher to keep the score of the game. The teams must take turns giving a number. The first team to successfully make a row, column, or diagonal total 15 wins. If neither team succeeds in making a total of 15, the game goes to the "cat".

In beginning the game, allow the students to use any numbers 1 through 9. Be sure to emphasize that once a number has been used, it cannot be used again during that game. Hopefully you will have some students discover that if they are first to place their number in the lattice, they can always win by placing a 5 in the middle. When this happens, change the numbers they are allowed to use to 0 through 9.

Again, they should soon discover that a 6 in the middle for the first team always produces a winner. You are now ready to change the rules for the final time. Allow one team only the even digits 0, 2, 4, 6, 8 and the other team the odd digits 1, 3, 5, 7, 9. You will find this makes for a completely fair and challenging contest. Encourage your students to play the game on their own.

VARIATIONS

This game should lend itself to more than just the whole numbers. For example, you might use it with only 0 and the negative integers -1 through -9. It might also be adapted for use of fractions or decimals.

SAMPLE GAME (Boys - even numbers, Girls - odd numbers)

<u>Choice</u>	<u>Boys</u>	<u>Girls</u>
1	0	3
2	4	5
3	8	7
4	6	1
5	2	

8	7	3
2	0	5
1	6	4

Game for the "cat"

CARD SORTING VIA BASE TWO

PURPOSE

To acquaint students with a functional use of base two. This particular "gimmick" shows how the binary system is used to solve a sorting problem.

METHOD

Obtain 15 IBM punch cards. On each card represent the numbers 0 through 15 and their equivalent base two notation.

(e.g.

0 = 0 ₂

1 = 1 ₂

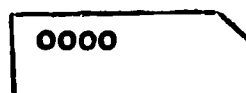
2 = 10 ₂

10 = 1010 ₂

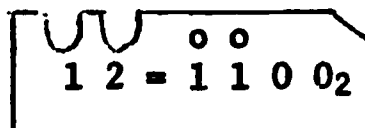
11 = 1011 ₂

etc.)

Next, align the cards in a pack and punch four holes in the positions indicated below:



Finally, cut out the space above each hole to represent a 1 in the base two number. Leave the space above the hole uncut to represent a 0. For example, the card representing 12 or 1100₂ would look like this:



When all of the cards have been punched and cut, shuffle them taking care to keep them all face up. (The cut corner on the IBM card will help you do this.) Then place a pencil through the hole on the right (units digit). Lift the pencil and you will note some of the cards will come up. Place these cards in front of the rest of the pack and repeat, this time placing the pencil in the second hole from the right. (The 2's place.) Continue this process through the hole until you have placed the pencil in each hole. When you have completed this process, you will note the cards are in order!

VARIATION

Have the students determine how many holes would be needed and how many cards would be needed to sort various numbers, e.g. only 5 holes would be needed to represent numbers up through 31, 6 holes would be needed to represent the numbers through 63, etc.

REFERENCE

Teaching General Mathematics, Max A. Sobel, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1967.

David R. O'Neil
11-67

THE MISER

PURPOSE

To develop interest in number patterns by the use of a math puzzle to be solved by the student.

METHOD

This particular "gimmick" involves a story which ends in the posing of a problem. It goes as follows:

There once lived a wealthy miser who had so much money he didn't know what to do with it. He finally settled on the idea of building his own vault and hiring an armed guard to watch the vault for him. When his vault was built, he gathered all of his money together in \$1000 bills and found he had a total of 32 bundles of one hundred \$1000 bills. To make sure that the guard didn't steal any of his money, the miser ordered him to place the bundles of money as in the figure below with a total of 9 stacks of money in each outside row.

1 7 1

7 7

1 7 1

However, the guard was able to come up with a scheme whereby he stole 12 bundles of \$1000 bills, 4 each on three different occasions. Each time the miser checked his money, he found no money missing since there were always 9 bundles of bills in each row.

Question: How was this possible?

VARIATION

Once you have determined the solution to this problem, you may be able to devise some similar problems of your own.

REFERENCES

The Boy's Own Book, 1863
Brain Resters and Testers, Cooperative Recreation Service, Inc.,
Delaware, Ohio.

(Answer to problem on reverse side)

David R. O'Neil
11-67

THE ONLY WAY TO RENT!

PURPOSE

The following story should be used as a source of amusement for students as well as a lesson in the importance of place value.

METHOD

The story goes as follows:

A landlord finally caught a tenant who was 7 weeks behind in his rent and threatened to evict him if he did not pay his back rent immediately. The landlord demanded a total of \$91 for the seven weeks' rent. He arrived at this figure by multiplying 7 weeks times the weekly rental rate of \$13 per week.

Without hesitation and rather indignantly, the tenant pointed out that he owed not \$91 but \$28. He then proceeded to prove his point as follows: "My rent is \$13 per week and I owe for 7 weeks, right?" The landlord agreed. "Then here is how I figure I owe you \$28. 3 times 7 is 21, and 7 times 1 is 7. 21 and 7 is 28." (See problem below)

The landlord wasn't taken in by this so the tenant continued. "To check multiplication we divide, right?" Again the landlord agreed. "OK, then we divide 7 into 28. 7 won't go into 2 but it will into 8; once. I put down 1, and subtract 7; that leaves me with 21. 7 goes into 21 exactly 3 times. So, 28 divided by 7 is 13. It checks".

The landlord was still skeptical so the tenant clinched his argument by adding a column of seven 13's as shown below. He added up the column of 3's and down the column of 1's and got a sum of 28 again. He paid his \$28 back rent and left the landlord scratching his head!!

$$\begin{array}{r} 13 \\ \times 7 \\ \hline 21 \\ 7 \\ \hline 28 \end{array}$$

$$\begin{array}{r} 13 \\ 7 \overline{)28} \\ \underline{7} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

$$\begin{array}{r} 13 \\ 13 \\ 13 \\ 13 \\ 13 \\ 13 \\ 13 \\ \hline 28 \end{array} \quad \begin{array}{l} \text{up} - 21 \\ \text{down} - \underline{7} \\ \text{sum} - 28 \end{array}$$

Note: The person who performs this trick must do so rapidly and aloud in order to make it effective.

REFERENCES

Brain Resters and Testers, Cooperative Recreation Service, Inc., Delaware, Ohio. You may have seen a similar routine done by the comedy team of Abbott and Costello.

David R. O'Neil
11-67

STEPPING THROUGH PAPER

PURPOSE

This is an exercise to both baffle and interest the student.

METHOD

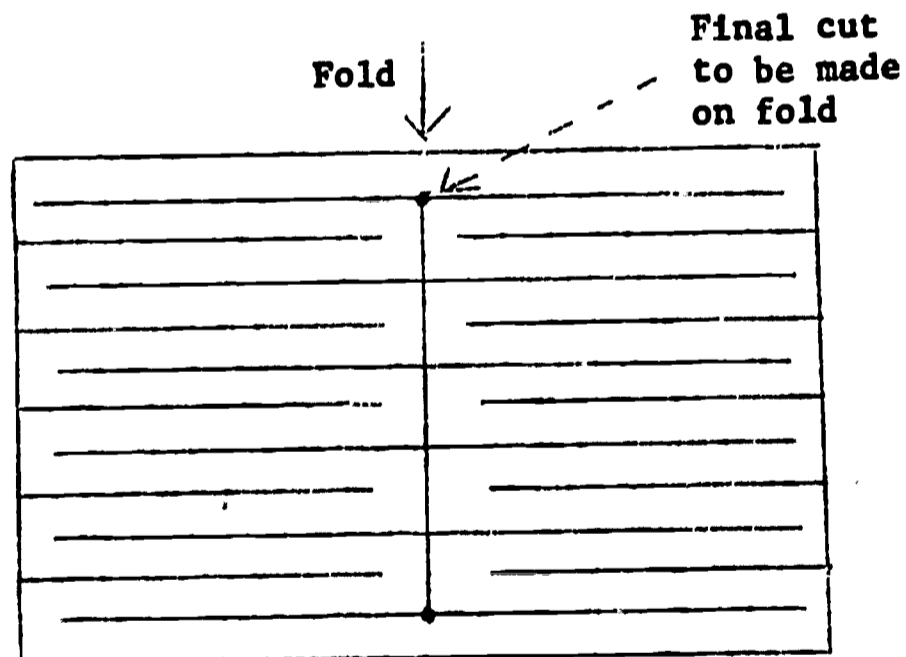
Take any piece of paper approximately the size of the standard $8\frac{1}{2}$ " x 11" stationery. Fold it lengthwise. Then cut the paper from the fold almost to the edge, every inch. Then turn the paper and cut BETWEEN these cuts, from lengthwise edges almost to the fold. Finally, cut along the fold from the first cross cut through the last cross cut as indicated below. When opened out, the piece of paper forms one large loop of sufficient size to pass over the body.

VARIATION

By cutting crosswise at smaller intervals, the loop can be made large enough to walk through!

REFERENCES

Cooperative Recreation Services, Inc., Delaware, Ohio.



David R. O'Neil

11-67